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DROUGHT, TREE RINGS, AND RESERVOIR DESIGN[1]

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ABSTRACT: Droughts constitute one of the most important factors affecting the design and operation of water resources infrastructure. Hydrologists ascertain their duration, severity, and pattern of recurrence from instrumental records of precipitation or streamflow. Under suitable conditions, and with proper analysis, tree rings obtained from long living, climate sensitive species of trees can extend instrumental records of streamflow and precipitation over periods spanning several centuries. Those tree-ring "reconstructions" provide a valuable insight about climate variability and drought occurrence in the Holocene, and yield long term hydrological data useful in the design of water infrastructure. This work presents a derivation of drought risk based on a renewal model of drought recurrence, a brief review of the basic theory of tree-ring reconstructions, and a stochastic model for optimizing the design of water supply reservoirs. Examples illustrate the methodology developed in this work and the supporting role that tree-ring reconstructed streamflow can play in characterizing hydrologic variability.
(KEY TERMS: drought; water supply; tree rings; water resources planning; stochastic models.)

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INTRODUCTION AND OBJECTIVES

Droughts, Humans, and the Environment

Droughts are insidious and pervasive. They cause substantial economic and environmental losses worldwide (Bryant, 1991). Their frequent and irregular occurrence has been a prime reason for the construction of water resources infrastructure intended to increase the reliability of water supplies in drought prone areas (Loáiciga and Renehan, 1997; Loáiciga *et al.,* 2000). The (natural) supply side, that is, precipitation, streamflow, or aquifer recharge, is an important factor involved in triggering droughts (Dracup *et al.,* 1980). The demand side is relevant also. Regions with large populations or intensive irrigated agriculture require a minimum level of water use to function adequately. Population growth and intensive use of water as a factor of production are conditions that accelerate the onset of (stressful) drought conditions. Drought conditions manifest themselves in various ways: reduced economic output and productivity, degraded environmental quality and aesthetics, and public indebtedness to finance short term, costly, substitute sources of water, to cite several common ones (Loáiciga and Renehan, 1997).

The Paradigm of Climate Prediction in Water Management

Due to the chaotic nature of the Earth's climate, it is not possible to predict the future climate accurately at regional scales with useful lead times, say, from a few years to a few decades ahead. Knowledge of the current climate in any region of the world, used in conjunction with complex general circulation models (GCMs) and regional climate simulation models (RCSMs), cannot be exploited to answer the question "when will the next drought occur, how long will it last, and how severe will it be?" (Loáiciga *et al.,* 1996; Mahlman, 1997; Grassi, 2000). Because of the impossibility of accurate future, long term drought prediction, water planners rely primarily on the study of the

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past climate to gain insights about the probable future climate. They analyze instrumental records of precipitation and streamflow – the two key variables in regional water planning – to extract from them information about the characteristics of droughts. In the absence of significant regional climatic change, it is reasonable to expect observed drought behavior to repeat itself – in a statistical sense – in the future (Loáiciga *et al.,* 1993). Therefore, so long as the historical precipitation and streamflow data sets have adequate temporal and spatial coverage, they constitute the main and best source of climatic information that water planners have at their disposal to plan, design, and manage water resources.

One word of caution is warranted concerning the use of historical hydrologic data in water resources planning. It is well established that the Earth's mean surface temperature has been increasing following the last glacial maximum 21,000 years ago (Clark *et al.,* 1999). Greenhouse emissions by human activities appear to have contributed to the natural warming trend, especially since the middle of the 18th Century with the onset of the Industrial Revolution. In spite of the uncertainty in long term future climate prediction, simulations of the Earth's mean surface temperature using leading GCMs suggest a 90 percent chance that it might increase 1.7 to 4.9°C from 1990 to 2100 (see, e.g., Karl and Trenberth, 2003, for a review of this matter). Deciphering how such increase in the mean planetary surface temperature might affect the incidence of drought at regional scales remains elusive (Loáiciga *et al.,* 1996). Several authors (see, e.g., Christensen *et al.,* 2004; Stewart *et al.,* 2004) contend that modern global warming is likely to impact snowpack and snowmelt dynamics in western North America in ways that could complicate water resources management, and, in particular, increase drought vulnerability. It is known from historical records that the hydrologic cycle of the arid American West exhibits substantial interannual variability, drought occurrence, and persistence. Modern global warming, it appears, may compromise the ability to deal with water scarcity in the face of growing human use of freshwater. Loáiciga *et al.* (1996) provide practical adaptive responses in water resources management to cope with potentially heightened hydrologic uncertainty in a warmer Earth.

The specter of modern global warming notwithstanding, this work adopts the paradigm "for water planning, the past is a clue to the future" as a working hypothesis, within which tree-ring based hydrologic reconstructions take preponderant relevance. In suitable climates – semi-arid and temperate regions of the western United States are a case in point – there are long lived species of trees whose annual growth is recorded and distinguishable as concentric annuli (called "rings" in the vernacular) in their mainstem tissue. The rings' interannual thicknesses depend, among other factors, on the amount of surface moisture – as precipitation or streamflow – a prime indicator of water and nutrient availability for biomass production. The precipitation versus tree-ring growth relationship is the key to precipitation reconstruction via tree rings. Tree rings may date back hundreds of years, even millennia, thus allowing streamflow reconstructions that may predate instrumental records by several centuries or millennia. This provides a widened window to past regional climate, and, with it, the possibility for a better characterization of droughts and of the risk that they pose to water management. The section "Tree Rings and Droughts" briefly reviews the basic methodology employed to make statistical inferences about past streamflow (or precipitation) based on tree rings.

Objectives

This paper reviews the basics of streamflow reconstruction using tree-ring data from long living species of trees sensitive to climatic fluctuations. In addition, the paper presents a derivation of the drought risk based on a renewal model of drought recurrence, and a stochastic optimization model for reservoir design. Tree-ring reconstructed streamflows are used in improving the characterization of drought risk and probability constraints in reservoir design. The probabilistic model of drought occurrence generalizes the renewal process introduced by Loáiciga *et al.* (1993) and Loáiciga and Leipnik (1996). The stochastic reservoir model introduces the technique of bootstrapping to characterize the probabilistic nature of cumulative streamflow in dealing with probabilistic constraints, and relies on long, tree-ring reconstructed time series to estimate the probability distributions of cumulative annual streamflows.

A DEFINITION OF DROUGHT

Drought is defined in this work as an extended period of low regional streamflow during which the natural water supply is not sufficient to meet normal water needs. Low streamflow may be defined in terms of below average, below median, or some other appropriate quantile. Regional streamflow encompasses the interaction between precipitation and evapotranspiration at regional scales. It feeds water supply reservoirs and sustains riverine ecosystems. It is positively correlated with precipitation, so that years with below median (above median) streamflow also exhibit less than normal (larger than normal) precipitation, and, consequently, reduced (increased) ground water recharge and soil moisture. An "extended period" is one that causes stress to human and environmental water use, and it is meaningful only in the context of specific water supply and water use conditions in a given regional setting. In regions with multi-year reservoir carryover capacity, three or more years of below median streamflow typically give rise to stressful water use. Many parts of California and the semiarid western United States exhibit a drought threshold of three or more years of below median streamflow (Loáiciga *et al.,* 1992, 1993; Loáiciga and Leipnik, 1996). The drought threshold can be shorter, however. Such is the case in regions with poorly diversified water supply sources. A case in point is the Edwards Aquifer of south-central Texas, one of the largest sole source aquifers in the United States (Loáiciga *et al.,* 2000).

There are several drought indicators in common use. One of them is the Palmer Drought Severity Index (PDSI) (Palmer, 1965; Karl, 1983, 1986), which measures the extent of dry weather. The PDSI is frequently used in the assessment of drought hazard for crop production. The definition of drought used herein, on the other hand, is tailored for the analysis, planning, design, and management of regional water supply systems with interannual storage carryover capacity.

The Drought Risk: Theory and Application

Droughts exhibit peculiar probabilistic characteristics (Sen, 1980; Loáiciga and Leipnik, 1996; Chung and Salas, 2000). Suppose that annual streamflow follows a probability density function (pdf) f(q). Let q_p denote the pth quantile of f(q), in which q_p could denote the median, for example (with p = 0.50), or some other level of streamflow selected to characterize drought occurrence. Furthermore, suppose that a drought occurs whenever there are θ or more consecutive years whose annual streamflows are less than q_p. Following Loáiciga *et al.* (1992, 1993) and Loáiciga and Leipnik (1996), the pdf of the duration D (years), f_D, of a drought is modeled as a truncated exponential distribution with (shape) parameter a_1

$$f_D(x) = a_1\, e^{-a_1(x-\theta)} \qquad x \geq \theta,\ a_1 > 0 \tag{1}$$

The interarrival time (T) (in years) elapsed between the ending of a drought and the beginning of the next drought is a random variable whose pdf is exponential with parameter a_2 (Loáiciga *et al.,* 1992, 1993; Loáiciga and Leipnik, 1996).

$$f_T(x) = a_2\, e^{-a_2 x} \qquad x \geq 0,\ a_2 > 0 \tag{2}$$

The sum of the duration of a drought plus the subsequent interarrival time is the renewal time (R) (in years), that is, R = D + T. Evidently, the renewal time is the time elapsed between the end of a drought and the end of the next one. Its expected value is a measure of the probabilistic regularity with which droughts recur. The pdf of the renewal time, $f_R(x)$, can be obtained from the pdfs $f_D(x)$ and $f_T(x)$ presented in Equations (1) and (2), respectively, using the facts that R = D + T and that D and T are independent random variables. Generalizing the results of Loáiciga and Leipnik (1996) from discrete to continuous pdfs, the pdf of the renewal time is as follows (for $a_1 \neq a_2$, the case of practical interest, although the case $a_1 = a_2$ is also tractable)

$$f_R(x) = \frac{a_1 a_2}{a_1 - a_2}\left[e^{-(x-\theta)a_2} - e^{-(x-\theta)a_1}\right] \qquad x \geq \theta \tag{3}$$

The expected value of the renewal time ($\bar{R}$) equals the sum of the expected values of D and T, so that $\bar{R} = \theta + 1/a_1 + 1/a_2$. The expected renewal time is the (average) recurrence interval between droughts.

The drought risk in a time interval $[\theta, t]$, Φ_t, equals the probability $P[R \leq t]$, which, using Equation (3), is

$$\Phi_t = \frac{a_1 a_2}{a_1 - a_2}\left[\frac{1 - e^{-(t-\theta)a_2}}{a_2} - \frac{1 - e^{-(t-\theta)a_1}}{a_1}\right] \qquad t \geq \theta \tag{4}$$

Figure 1 shows the historical (instrumental) time series of annual streamflow in the upper Santa Ynez River (at Bradbury Dam, Cachuma Reservoir), Santa Barbara County, California, from 1917 through 2000 (Loáiciga, 2002). The numbers 1 through 8 in Figure 1 identify the droughts that occurred between 1917 and 2000, in which the defining criterion for drought was a run of three or more years with below median annual streamflow (see Loáiciga, 2002, for further justification of this criterion). The median annual streamflow is shown as a horizontal line in Figure 1, and equals 35.2 million cubic meters. The historical time series shown in Figure 1 was pooled with a time series of tree-ring reconstructed annual streamflow for the upper Santa Ynez River from 1401 through 1916 (Loáiciga *et al.,* 1993; Loáiciga and Leipnik, 1996), to produce a 600-years long pooled time series. The pooled (tree-ring based and historical) time series tested positively for statistical stationarity, and tree-ring reconstructed streamflow preserved the serial correlation and variance characteristics of the historical streamflows (Loáiciga *et al.,* 1993; Loáiciga and Leipnik, 1996).

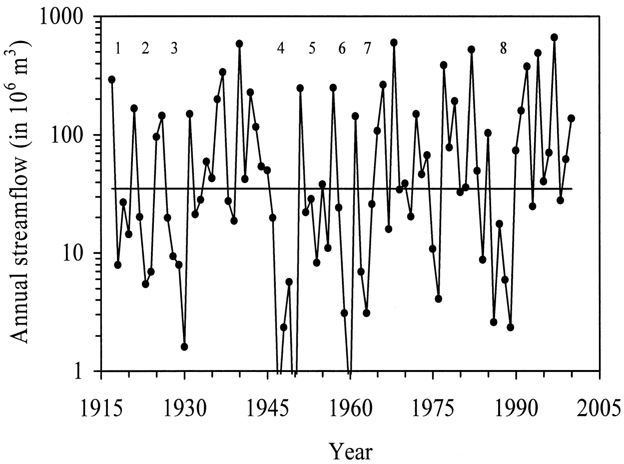

Figure 1. Mean Annual Streamflow in the Upper Santa Ynez River at Bradbury Dam. Droughts are numbered 1 through 8. The median equals 35.2 x 10^6 m^3/yr and is shown as a horizontal line.

There were 43 droughts between 1401 and 2000 in the upper Santa Ynez River, for an average drought interarrival time of approximately 14 years. Drought recurrence was modeled according to the renewal model embodied by the pdf in Equation (3) to calculate the drought risk (see Equation 4). The exponential pdfs (Equations 1 and 2) and the renewal time pdf (Equation 3) were tested statistically against the 1401 to 2000 streamflow and drought data using the chi-squared goodness-of-fit test, whose results validated the proposed pdfs. The estimates of the parameters a_1 (in Equation 1) and a_2 (in Equation 2) were 1.0 and 0.10, respectively. Using these estimates with the threshold parameter θ = three years, the drought risk (see Equation 4) was calculated and plotted as a function of the time horizon (t) ranging from θ through 1,000 years in Figure 2.

It is seen in Figure 2 that the probability of at least one drought θ = three years long or longer is approximately 50 percent in any ten-year period, and that it equals one in any period 50 years long or longer. This level of drought risk suggests provision of sufficient reservoir storage to achieve reliable water supply, as demonstrated later in this article. One of the rewards of extending streamflow (or precipitation) records by means of accurate tree-ring reconstructions is the improved estimation of the pdfs of the drought duration, the interarrival time, the renewal time, and of the drought risk in an interval of specified length with records much longer than historical ones. Furthermore, optimization models for reservoir design can be constructed based on the tree-ring extended time series, as shown in an example below.

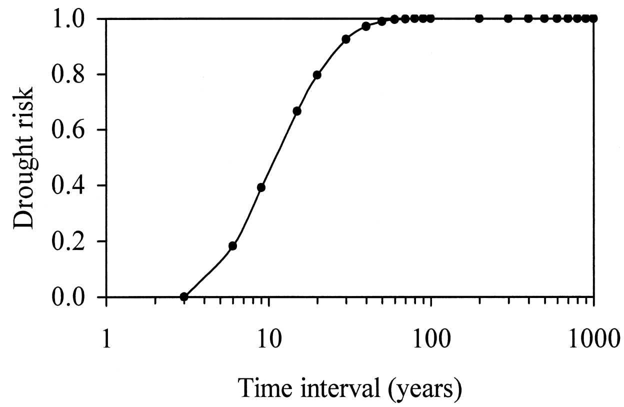

Figure 2. The Drought Risk as a Function of the Time Interval [θ, t].

TREE RINGS AND DROUGHTS

Tree-ring based hydrologic reconstructions rely on trees whose tree-ring growth is sensitive to precipitation – or its proxy, streamflow – and other ambient variables, surface temperature among them. Time series of tree-ring growth are used to reconstruct the long term behavior of important hydrologic variables such as streamflow and precipitation. A particular form of the transfer equation relating, say, streamflow in a time interval t, denoted by Q_t ($10^6 m^3$), the dependent variable), to a set of tree-ring widths, Y (microns) the predictor variable, is as follows

$$Q_t = \sum_{j=1}^{J} \sum_{k=0}^{K} b_{j,k} \ Y_{j,t+k} + v_t \tag{5}$$

in which $b_{j,k}$ is the regression coefficient corresponding to the jth tree-ring sampling site at time t + k, and v_t is a zero mean, constant variance, error term.

The tree-ring widths in Equation (5) can have time shifts (or lags) of up to size K (years), implying that the current value of the dependent value (Q_t) is regressed on the current and a small number of future values of the tree-ring width. Some authors have included negative lags (i.e., k < 0) in Equation (5) (see Stockton and Meko, 1983) and additional predictor variables, such as wood density, besides tree-ring width (see Loáiciga *et al.,* 1993, for a review of this subject). The lagged structure of Equation (5) reflects the multi-year effect that hydroclimatic forcing may have on tree growth. That persistent forcing is attributed to the physiologic conditioning that hydroclimatic fluctuations exert on food storage, crown area, and the root mass of trees (Fritts, 1976; Cleveland and Stahle, 1989).

The b coefficients in Equation (5) are estimated using contemporaneous time series of tree-ring widths and historical records of streamflow (or precipitation, or the PDSI, to cite a few dependent variables). The estimation of the b coefficients has been tackled with a variety of statistical and empirical fitting techniques, such as regression analysis and polynomial fitting, aided by principal component analysis (PCA) and canonical correlations in some instances (Jones *et al.,* 1984). Following this calibration phase, long term tree-ring time series predating the period of instrumental hydrologic recording are used to reconstruct annual time series of the dependent variable Q_t using the estimated form of the transfer Equation (5). To illustrate, in the case of a predictor variable sampled at two locations, and with one time lag, the estimated equation used in hydroclimatic reconstruction would be

$$\hat{Q}_t = \hat{b}_{1,0} Y_{1,t} + \hat{b}_{1,1} Y_{1,t+1} + \hat{b}_{2,0} Y_{2,t} + \hat{b}_{2,1} Y_{2,t+1} \qquad (6)$$

in which the "hat" signifies an estimate.

Verification of the predictive skill of the estimated equation can be performed with a variety of goodness-of-fit statistics, using persistence coefficients (Hurst coefficient), and cross validation, to cite common schemes found in the literature (see Loáiciga *et al.*, 1993, for a long list of references on this subject). The pathway to a calibrated and verified estimated equation with acceptable predictive skill is arduous. Many confounding issues need be resolved prior to achieving a relatively accurate tree-ring reconstruction of the dependent variable Q. The predictor variables (Y) in Equations (5) or (6) represent averages (or medians or modes) of tree-ring widths belonging to many trees found at a sampling location. The raw ring-width data are detrended and standardized to dimensionless variables called tree-ring indices prior to averaging. One common approach to standardization is to divide a given year's ring width by that width that would be expected from a normal decline of ring width in trees caused by aging. The age induced ring widths impose a growth trend that must be removed to separate the growth due to climate forcing from other biologically driven growth. Techniques for fitting the age driven growth and for standardization are reviewed in Cook (1987). The removal of the age induced growth trend filters out low frequency (that is, long period), climatically induced variability in the tree-ring series. It is not straightforward to determine how much of the legitimate climatically induced variability in the tree-ring series is removed by detrending and standardization. Concerning this issue, some authors (see Michaelsen *et al.,* 1987) claim that most of the variability on time scales shorter than 100 years is retained.

Serial and Spatial Correlation

Other issues that influence the statistical analysis of the tree-ring indices prior to their use in hydroclimatic reconstructions are serial and spatial correlation. Serial correlation is due to the dependence of a tree's present growth on its growth in previous seasons. The serial (or temporal) correlation in the tree-ring indices is commonly removed by subtracting the identified autoregressive component (if any) from the tree-ring indices. Detrending, standardization, and removal of serial correlation from a tree-ring series produce a pre-whitened time series of tree-ring indices, known as a tree-ring chronology. Some authors also remove the autoregressive component of the streamflow time series prior to calibration of the transfer Equation (5). The autoregressive component is added later to produce a reconstructed streamflow with statistical resemblance to the historical streamflow. The removal of serial correlation in the tree-ring or streamflow data is a matter of choice. Model calibration, statistical estimation, and hydroclimatic reconstruction are possible with serially correlated data using statistical techniques that are available for that purpose.

Regional streamflow and tree-ring data exhibit strong patterns of statistical dependence. Because of the strong spatial correlation, tree-ring data at one sampling location can explain a high percentage of the variance at other locations. The inclusion of a set of spatially correlated data from all the sampling locations into a transfer (or regression) equation relating them to streamflow results in an ill conditioned regression with meaningless coefficients, the so called multicollinearity problem. This problem can be overcome by choosing subsets of data from the complete set of correlated variables from which multicollinearity is removed. Principal component analysis and canonical correlation analysis (CCA) are two techniques widely used to cope with multicollinearity. To illustrate, suppose that PCA is applied to spatially correlated tree-ring data (denoted by the predictor variable (Y in Equation 5) to reconstruct streamflow. Suppose, furthermore, that there are N sampling stations at which annual ring widths are measured. Then, there are N principal components of the tree-ring data. The first principal component is the linear combination of the N ring widths that explains most of the variation in ring widths from site to site. A linear combination of the N ring widths is the weighted sum $c_1Y_1 + c_2Y_2 + \ldots + c_NY_N$, in which the weights ci are calculated by PCA. The second principal component has the largest variance among all linear combinations of the N site ring widths uncorrelated with the first principal component. In general, the ith principal component explains most of the variance among all the linear combinations of the N site ring widths uncorrelated with the first, second, …, (i-1)th principal components. From the PCA, the hydrologist retains – for reconstruction purposes – those principal components that explain most of the tree-ring width variance. Those are the predictor variables (Y) used in Equation (5). Further discussion of PCA (and of CCA as well) can be found in Loáiciga *et al.* (1993), along with some examples of tree-ring reconstructions.

There are other complicating phenomena in tree-ring reconstructions. Forest fires, pests, and disease that weaken trees and affect tree-ring growth are examples. The identification of hydroclimatic forcing in tree rings may require substantial collateral analysis. The Tree Ring Laboratory of the University of Arizona (Tucson) and the National Climatic Data Center (2000) have produced several tree-ring chronologies for various regions of the United States potentially useful for hydroclimatic reconstructions.

A STOCHASTIC MODEL FOR RESERVOIR DESIGN

This section presents a stochastic model for reservoir design in which long term, accurate, streamflow time series are particularly useful. The model can be used with or without tree-ring reconstructed data, but, accurate, long term, tree-ring reconstructions can be effectively integrated in the modeling effort. The reservoir model's objective function is the minimization of the total present cost of building a reservoir capacity C (10^6m^3) (this is a positive cost), minus the revenue stemming from water releases from the reservoir, which meet beneficial functions. The model's decision variables are the reservoir capacity (C) and the releases x_i (10^6m^3), i = 1, 2, …, n during the operation horizon. The constraints on the decision variables are: (1) the probability that the reservoir storage be less than the reservoir capacity in year i, i = 1, 2, …, n, must be at least α, in which α is close to one; (2) the probability that the storage capacity be at least zero must be equal to or larger than α in each year of operation (1, 2, …, n); and (3) reservoir releases (annual values) must be at least equal to a desired target (annual) value (F in 10^6m^3). The first two sets of constraints on reservoir storage are probabilistic and must be converted to their deterministic equivalents prior to the solution of the optimization model. The model considers the effect of evaporation from the reservoir and precipitation onto the reservoir. Following Loáiciga (2002), the evaporation during the ith year (E_i in 10^6m^3) is equal to the reservoir unit evaporation (e_i in meters) times the average reservoir surface area during that year, calculated as the arithmetic mean of the beginning of year area (A_{i-1} in 10^6m^2) and the end of year reservoir area (A_i in 10^6m^2). The area (A in 10^6m^2) versus storage (S in 10^6m^3) relationship is approximately linear, $A = a + b\,S$, in which the coefficients a, b must be estimated from area and storage data. Likewise, the precipitation during the ith year (P_i in 10^6m^3) is the product of unit precipitation onto the reservoir (p_i in m) times the average reservoir area during that year.

The objective function of the optimization model is, in which G is the unit cost of building reservoir capacity (US\$/10^6m^3), W_i is the unit net revenue from annual reservoir releases (US\$/10^6m^3), and s is the discount rate) (dimensionless)

$$\min_{\substack{\text{w.r.t.}C,\\ x_i, i=1,2,\ldots,n}} \quad G \cdot C - \sum_{i=1}^{n} \frac{1}{(1+s)^i} W_i \cdot x_i \tag{7}$$

The objective function is subject to the following constraints

$$P(S_i \le C) \ge \alpha \qquad i = 1, 2, \ldots, n \tag{8}$$

$$P(S_i \ge 0) \ge \alpha \qquad i = 1, 2, \ldots, n \tag{9}$$

$$x_i \ge F \qquad i = 1, 2, \ldots, n \tag{10}$$

The decision variables C and x_i are nonnegative.

To bring the optimization model, Equations (7) to (10), to solvable form, the probabilistic constraints, Equations (8) and (9), must be converted to their deterministic equivalents. To that end, it is useful to write the dynamic equation of reservoir storage in terms of the previous storage (S_{i-1}) and fluxes in and out of the reservoir (those are: known diversions from the reservoir, D_i; evaporation, E_i; precipitation, P_i; streamflows, Q_i; and water releases, x_i)

$$S_i = S_{i-1} + Q_i + P_i - D_i - E_i - x_i \qquad 1 = 1, 2, \ldots, n \tag{11}$$

Equation (11) is initialized with S_0, the starting storage, which is specified by the analyst as a fraction (g, $0 \le g \le 1$) of the reservoir capacity. Typically, the starting storage is set equal to one half of the reservoir capacity, $S_0 = 0.5$ C. The evaporation $E_i = 1/2\ (A_{i-1} + A_i)\ e_i$ and the precipitation $P_i = 1/2\ (A_{i-1} + A_i)\ p_i$. The unit reservoir evaporation (e_i) and precipitation (p_i) are data that must be specified by the analyst. If not available for the entire horizon i = 1, 2, …., n, they can be replaced by their averages calculated from instrumental records without great effect on the model's solution. The surface area of the reservoir and the reservoir storage are related by the equation $A_j = a + b\ S_j$, j = i-1 or i. Successive substitution of the beginning of period storage in Equation (11) leads to the formulation of the storage equation in terms of the initial storage (S_0 = g C)

$$S_i = \psi_i gC + \sum_{k=1}^{i} T_k \phi_k Q_k - \sum_{k=1}^{i} T_k \phi_k D_k - \sum_{k=1}^{i} T_k \phi_k x_k + \sum_{k=1}^{i} G_k \phi_k \qquad (12)$$

for i = 1, 2, …, n, in which

$$\psi_i = \prod_{k=1}^{i} \left(\frac{1 + (p_k - e_k)\frac{b}{2}}{1 - (p_k - e_k)\frac{b}{2}} \right) \qquad (13)$$

$$\phi_k = \prod_{r=k+1}^{i} \left(\frac{1 + (p_r - e_r)\frac{b}{2}}{1 - (p_r - e_r)\frac{b}{2}} \right) \quad k = 1, 2, \ldots, i; \textit{with } \phi_i = 1 \qquad (14)$$

$$G_k = \frac{a(p_k - e_k)}{1 - (p_k - e_k)\frac{b}{2}} \qquad (15)$$

$$T_k = \frac{1}{1 - (p_k - e_k)\frac{b}{2}} \qquad (16)$$

Equation (12) is substituted in the probabilistic constraints, Equations (8) and (9). The deterministic equivalent of Equation (8) becomes (with S_0 = g C)

$$(1 - g\psi_i)C + \sum_{k=1}^{i} T_k \phi_k x_k \geq q_\alpha^{(i)} - \sum_{k=1}^{i} T_k \phi_k D_k + \sum_{k=1}^{i} G_k \phi_k \qquad (17)$$

for i = 1, 2, …, n, and that of Equation (9) is (with S_0 = g C)

$$-g\psi_i C + \sum_{k=1}^{i} T_k \phi_k x_k \leq q_{1-\alpha}^{(i)} - \sum_{k=1}^{i} T_k \phi_k D_k + \sum_{k=1}^{i} G_k \phi_k \qquad (18)$$

for i = 1, 2, …, n. The quantiles $q_\alpha^{(i)}$ and $q_{1-\alpha}^{(i)}$ appearing in Equations (17) and (18) are defined by the following probabilistic statements

$$P\left[\sum_{k=1}^{i} T_k \phi_k Q_k \leq q_\alpha^{(i)} \right] = \alpha \qquad (19)$$

$$P\left[\sum_{k=1}^{i} T_k \phi_k Q_k \leq q_{1-\alpha}^{(i)} \right] = 1 - \alpha \qquad (20)$$

$q_{1-\alpha}^{(i)}$ and $q_\alpha^{(i)}$ are the 1-α and α quantiles, respectively, of a linear combination of i streamflows.

The bootstrapping method (Efron, 1983) is well suited for the estimation of those quantiles, which requires a sufficiently long time series of streamflows. Long time series (i.e., spanning several centuries) from tree-ring reconstructions are particularly useful in this regard. The bootstrapping method adapted to the optimization model is described by the following computational algorithm: (1) for i = 1, use the (estimated) pdf of the entire streamflow time series to estimate $q_{1-\alpha}^{(i)}$ and $q_\alpha^{(i)}$; (2) for i = i + 1, draw randomly (with replacement) from the time series of n streamflows a large number M of samples (e.g., M = 200) each containing i streamflow values and for each sample calculate the following linear combination of the streamflow values (q in $10^6 m^3$)

$$\sum_{k=1}^{i} T_k \phi_k q_k \qquad (21)$$

(3) fit a theoretical pdf (gamma, lognormal, or other) to the M values of the linear combinations written in Equation (21), test the pdf with a goodness-of-fit test (e.g., chi squared), and estimate $q_{1-\alpha}^{(i)}$ and $q_\alpha^{(i)}$; and (4) repeat Steps (2) and (3) until i = n. At this point, the deterministic constraints in Equations (17) and (18) can be stated numerically and the linear programming problem whose objective function is described in Equation (7), subject to the constraints of Equations (17), (18) and (10), can be solved.

Example of Reservoir Design

The reservoir design model presented above was applied to the upper Santa Ynez River basin of California. Loáiciga (2002) described the upper Santa Ynez River basin from a water resources viewpoint, in conjunction with the development of a nonstochastic reservoir design model. The stochastic model of this section extends the work of Loáiciga (2002). The data used in the stochastic model were: (1) g = 0.5, thus, the initial storage was set equal to one half of the (unknown) reservoir capacity; (2) the annual downstream water requirement F = 2.932 x 10^6m^3 and the annual reservoir diversion (constant) D_i = D = 39.825 x 10^6m^3; (3) the cost factors G = US$ 1.3 x 10^6 per 10^6m^3 of reservoir capacity and (constant) W_i = W = US$ 1.3 x 10^6 per 10^6m^3 of reservoir release; (4) a = 1.3007, b = 0.05054 in the area (in 10^6m^2) versus storage (in 10^6m^3) equation; (5) n = 84, the number of years in the optimization horizon (1917 to 2000); (6) the discount rate s = 0 (i.e., no discounting); (7) reservoir evaporation (e_i) and precipitation (p_i) data from Loáiciga (2002); and (8) the distributions of the sums of annual streamflows were derived using the bootstrapping method described above, in which the historical record of annual streamflow in the upper Santa Ynez River (1917 to 2000, see Figure 1) was pooled with a 1401 to 1916 tree-ring reconstruction of streamflow for that same river (introduced above, see Loáiciga *et al.,* 1993; Loáiciga and Leipnik, 1996). This produced a 600-year long streamflow data set used to bootstrap the distributions of the sums of streamflows that appear in the optimization model.

Figure 3 shows the optimized reservoir capacity as a function of the probability level α ranging from 0.75 to 0.99. The graph in Figure 3 indicates that as the probability level (also called reliability) becomes closer to 1, the reservoir capacity increases in a nonlinear fashion. The increasing trend in reservoir capacity with increasing reliability reveals that as the uncertainty of undesired reservoir performance is decreased – in this case by increasing the probability in the model's constraints – the size of the reservoir must increase. In other words, there is an increasing premium associated with the reduction of the likelihood of poor performance.

To put the results of Figure 3 in perspective, the reservoir capacity was determined using the (deterministic) mass curve method (see, e.g., Linsley and Franzini, 1979). Figure 4 shows the mass curve for the upper Santa Ynez River. The mass curve is the cumulative annual streamflow plotted as a function of time. The dotted line shown as a tangent to the mass curve has a slope of 52 x 10^6m^3/yr, which equals the annual demand of Santa Ynez River water (including evaporative losses). The maximum vertical departure between a tangent to the mass curve and the mass curve equals the necessary reservoir capacity to meet a specified annual demand of river water. That reservoir capacity is represented by the vertical line in Figure 4, directly above 1950, and equals 290 x 10^6m^3, the design capacity of the existing Cachuma Reservoir behind Bradbury Dam in the upper Santa Ynez River.

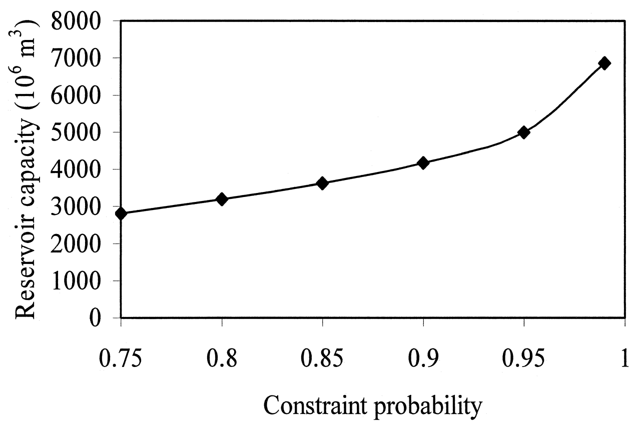

Figure 3. Reservoir Capacity Versus the Constraint Probability (α) Obtained With the Optimization Model.

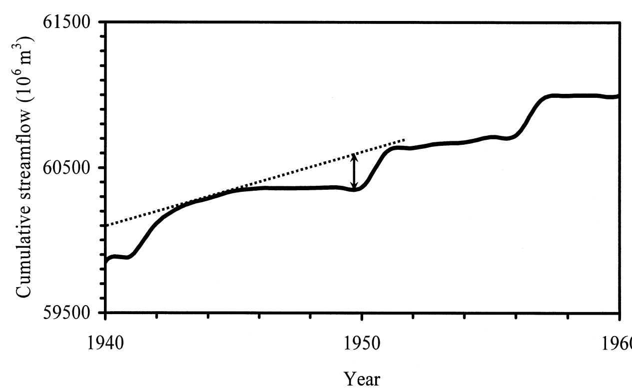

Figure 4. Mass Diagram and Reservoir Capacity Determination (290 x 10^6m^3), Upper Santa Ynez River at Bradbury Dam, California.

The existing reservoir capacity is about one tenth of the reservoir capacity found with the stochastic reservoir model for a probability level α = 0.75 (see Figure 3). Only that portion of the mass curve between 1940 and 1960 is shown in Figure 4 to attain adequate visual resolution. The entire mass curve, from 1401 through 2000, underwent the same type of analysis shown in Figure 4 in search of the required reservoir capacity. Fortuitously, the critical period within which the mass curve reservoir capacity lies is

1940 to 1960. Pooling of the tree-ring reconstructed streamflow (1401 to 1916) with the historical streamflow time series changed the level of the mass curve, but it did not alter the maximum vertical departure between tangents to the mass curve and the mass curve itself.

The model optimized reservoir capacities (see Figure 3) are at least one order of magnitude larger than the mass curve determined capacity, which happens to be the size of Cachuma Reservoir in the upper Santa Ynez River. Evidently, the stochastic treatment of reservoir inflows and the imposition of probabilistic constraints call for reservoir capacity larger than that obtained with the mass curve method (which treats reservoir inflows deterministically and does not impose explicit constraints). This is consistent with previous research pointing to the conservativeness of reservoir capacity determination using probabilistically constrained methodology such as that presented above (see, e.g., Loáiciga, 1988). The performance of the existing Cachuma Reservoir in the upper Santa Ynez River has shown that additional reservoir capacity is indeed needed to meet fisheries and water supply requirements with acceptable reliability. In fact, the steelhead trout population (*Oncorhynchus mykiss*) has dwindled to near extinction downstream from the Cachuma Reservoir due to insufficient reservoir releases. Moreover, several cities depending on upper Santa Ynez River streamflow have had to search for additional water sources to mitigate shortages caused by frequent droughts. Those sources include interregional water imports and desalination of ocean water (Loáiciga and Renehan, 1997). Disputes over the environmental impacts of raising Bradbury Dam to enlarge Cachuma Reservoir have effectively prevented the development of additional storage capacity.

CONCLUSION

This paper has: (1) reviewed methodology for tree-ring reconstructions of streamflow; (2) developed a formula to quantify drought risk based on the renewal model of drought recurrence, and tested the formula with pooled historical and tree-ring reconstructed streamflow time series; and (3) developed a stochastic model for reservoir design in which bootstrapping of flow quantiles was aided by pooling tree-ring reconstructed and historical streamflows. Two examples of the methodology presented in this article demonstrated that reconstructed streamflows featuring good predictive skill are valuable in assessing the probabilistic nature of drought recurrence and improving the parameterization of stochastic reservoir models.

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