Multireservoir Multiyield Model with Allowable Deficit in Annual Yield

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Abstract: Yield models produce reasonable reservoir designs with release reliabilities near targets. This paper extends the basic yield model and presents a multiple-yield model for a multiple-reservoir system consisting of single-purpose and multipurpose reservoirs. The objective is to achieve prespecified reliabilities for irrigation and energy generation and to incorporate an allowable deficit in the annual irrigation target. A single-reservoir yield model illustrates how a single-yield problem can be converted to a multiple-yield problem that represents the same irrigation deficit criterion while maintaining the desired reliability. The yield model is applied to a system of eight reservoirs in the upper basin of the Narmada River in India. The results are analyzed for four cases. As the desired reliabilities for yields can be prespecified for different purposes by consideration of an allowable deficit in annual yield in a multiple-reservoir system, this model can act as a better screening tool in planning by providing outputs that can be very useful in improving the efficiency and accuracy of detailed analysis methods such as simulation.

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Introduction

A yield model is an implicit stochastic linear programming (LP) model that incorporates several approximations to reduce the size of the constraint set needed to describe reservoir system operation and to capture the desired reliability of target releases considering the entire length of the historical flow record. The yield model estimates over-year and within-year reservoir capacity requirements separately to meet the specified release reliability targets. Over-year capacity is governed by the distribution of annual streamflows and the annual yield to be provided. The maximum of all over-year storage volumes is the over-year storage capacity. Any distribution of within-year yields that differs from the distribution of the within-year inflows may require additional active reservoir capacity. The maximum of all within-year storage volumes is the within-year storage capacity. The total active reservoir storage capacity is simply the sum of the over-year storage and within-year storage capacities.

The concept of a yield model was introduced by Loucks et al. (1981); Stedinger et al. (1983) reviewed and compared deterministic, implicitly stochastic, and explicitly stochastic reservoir screening models. The models were applied to a hypothetical three-reservoir water supply problem, and results were compared with simulation. Both Loucks et al. and Stedinger et al. concluded that (1) simple screening models that can identify potentially efficient system designs are highly desirable; (2) purely deterministic screening models based on historical mean monthly flows do not provide sufficient reservoir capacity to achieve target reliabilities; (3) use of the most critical flows in a record leads to larger reservoir capacities and higher system reliabilities; (4) the explicit storage models, linear-decision rule, chance-constrained formulations of Revelle et al. (1969) and Loucks (1970) overestimated reservoir capacity and generated operating policies that failed to utilize available water and storage space efficiently; and (5) the yield model of Loucks et al. (1981) produced reasonable reservoir designs with release reliabilities near targets.

The concept of the original LP-based yield model is employed through a combination of simulation and nonlinear optimization techniques for multipurpose, multireservoir systems (Lall and Miller 1988; Lall 1995; Sinha et al. 1999). Reservoir capacities are determined by using a new sequent-peak algorithm, and the annual yield reliability is considered a decision variable. These studies considered the annual deficit in irrigation by specifying the degree of yield failure. However, the dependence of reservoir capacities on monthly inflows and releases is evaluated independently using monthly simulations with respect to annual target yields. The nonlinear optimization model considers only annual releases for different purposes. In the linked simulation-optimization formulation, mass balance equations and the decision variables, such as release and storage, are not explicitly considered but are implicitly satisfied through the simulation. Dandy et al. (1997) made a comparison of simulation, network linear programming, the full optimization model, and the LP yield model for estimating the safe yield of the Canberra water supply system, consisting of four reservoirs.

Dandy et al. (1997) pointed out that, although a simulation model will accurately assess the system yield for an assumed set of operating rules, it will not assess the maximum yield that can be achieved by adopting the best possible set of operating rules for the system. Optimization models have the advantage of not requiring that operating rules be specified; instead, optimum op-

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erating decisions are made at every time step in order to obtain the maximum yield in a single run without the need for an iterative procedure, as in the case of simulation models. Even though optimization techniques are likely to give relatively less accurate reservoir yield estimates than detailed simulation, these yield estimations can be useful in the planning of new resources, wherein accurate prediction of future yields can be difficult because of the lack of defined operating rules for the new system. Dandy et al. (1997), however, pointed out that if the system yield with a specified, less-than-maximum reliability needs to be determined, there is considerably more difficulty in using the optimization and yield models.

When the reservoir yield has a reliability lower than the maximum, the percentage of annual yield to be made available from the reservoir (or the allowable annual yield deficit) during failure years needs to be specified (Loucks et al. 1981). Stedinger et al. (1983) used this concept to define the allowable deficit in annual reservoir yield during the failure years. It is possible to specify a failure fraction to define the allowable deficit in annual reservoir yield during the failure years in a single-yield problem (single-purpose reservoir). For a multipurpose reservoir concerned with both irrigation and hydropower, one firm (safe) and one secondary yield with lower reliability can be defined. However, in such a case, it is not possible to define a failure fraction to be greater than zero for the secondary yield as the firm yield is essentially increased by an amount equal to the failure fraction times the secondary yield (Loucks et al. 1981).

The yield model can be extended to multisite problems. The essential requirement is that the annual reservoir yields should have the same reliability throughout the basin for maintaining continuity. This requirement can be satisfied if the multireservoir system is single purpose, as demonstrated by Stedinger et al. (1983) and Dandy et al. (1997). An allowable annual deficit criterion for the annual firm reservoir yield can also be incorporated in a single-purpose, multireservoir system (Stedinger et al. 1983). It is also possible to satisfy the reliability requirement in a multipurpose, multireservoir system with the same number of purposes (and yields) at each reservoir site and the same reliability for each purpose at all the reservoirs. However, an allowable annual deficit criterion for a purpose such as irrigation having less than the maximum annual yield reliability (represented by the annual secondary reservoir yield) cannot be incorporated (failure fraction has to be zero during failure years).

A multiple-reservoir system consisting of a combination of single-purpose irrigation, single-purpose hydropower, and multipurpose reservoirs to achieve prespecified annual reservoir yield reliabilities is considered in this study, which also incorporates an allowable annual deficit criterion for irrigation. The single-purpose hydropower and multipurpose reservoirs can be modeled as multiple-yield formulations incorporating annual firm and secondary reservoir yields. However, a single-purpose irrigation reservoir will have to be modeled by a single-yield formulation if an allowable deficit criterion is to be included. This poses a problem of maintaining the continuity of reservoir yields among different reservoir sites, as the hydropower and multipurpose reservoirs include multiple yields. If an irrigation reservoir is modeled with a multiple-yield formulation, the desired reliability of the annual yield for irrigation, which is less than the annual reliability for annual firm energy, cannot be obtained. Moreover, an allowable deficit criterion for irrigation (considered as annual secondary reservoir yield) cannot be incorporated in the multiple-yield formulation in the case of irrigation as well as multipurpose reservoirs.

This clearly means that if a multiple-reservoir system consists of a combination of single-purpose and multipurpose reservoirs, the yield model available in the present form cannot be applied. This paper extends the basic yield model to address these issues. A single-reservoir yield model is first presented to show how a single-yield problem can be posed as an equivalent multiple-yield problem representing the same allowable deficit criterion while maintaining the desired reliability. Then an application to a system of eight reservoirs in the upper basin of the Narmada River in India is presented to illustrate how the multiple-yield model can be applied to a multiple-reservoir system consisting of single-purpose irrigation, single-purpose hydropower, and multipurpose reservoirs. The conceptualization and details of the yield model on which the present model development is based are presented in Loucks et al. (1981, pp. 339–353, 368–371).

An earlier study conducted by Chaturvedi and Srivastava (1981) [see also Ministry (1965)] investigated alternative combinations, capacities, and operating policies of six major projects: Bargi, Tawa, Narmada Sagar, Harinphal, Jalsindhi, and Navagam (Sardar Sarovar). The study determined the optimum height of Sardar Sarovar, the terminal storage dam. In the study, deterministic linear programming models [linear programming deterministic continuous (LPDC) and linear programming deterministic discontinuous (LPDD)] were employed for screening, followed by simulation to decide the alternative combinations and capacities of these six major projects. The LPDC model regulated the mean monthly flows, whereas the LPDD model used wet and dry years in order to deviate from regulating mean monthly flows. The proposals for the Harinphal and Jalsindhi projects were later dropped by the planners, as not enough storage was available at these two sites to meet the irrigation demands for the optimum height of the Sardar Sarovar dam.

This paper considers the water resource potential of a system of eight reservoirs in the upper part of the Narmada River basin using a yield model. The Master Plan for the basin [Government (1972)] stipulates 29 major projects in the state of Madhya Pradesh out of the total of 30 major projects in the entire river basin. The terminal major project Sardar Sarovar lies in the state of Gujarat.

Single-Reservoir Yield Model

Single-Yield Formulation with Allowable Deficit Criterion

Consider a single-purpose reservoir i (subscript i is used for consistency in notation with the multiple-reservoir model presented later) of given capacity not affected by upstream regulations for which the annual yield with reliability lower than the maximum reliability is to be determined. A 20% deficit in annual reservoir yield is allowed during failure years. The annual yield having 70% reliability from a reservoir with a known capacity is to be determined. The single-yield model formulation (Loucks et al. 1981) for the problem is as follows. The firm annual and time (t) period yields with a reliability p are denoted by $Oy_i^{f,p}$ and $Oy_{f,p}^{i,t}$, respectively; $S_{i,j-1}^0$ and $S_{i,j}^0$ represent the initial and final over-year reservoir storage volumes in year j, whereas the initial and final within-year storage volumes in time period t are denoted by $S_{i,t-1}^w$ and $S_{i,t}^w$, respectively; $I_{i,j}$ is the annual inflow and $Sp_{i,j}$ is the annual excess release from the reservoir in year j with over-year and total active storage capacities Y_i^0 and Ya_i, respectively. The allowable annual yield deficit is represented by $(1-\theta_{p,j})$, where $\theta_{p,j}$ is the failure fraction defining the proportion of the annual reservoir yield to be made available during the failure years.

$$\text{Maximize} \quad Oy_i^{f,p} \tag{1}$$

Subject to

1. Over-year storage continuity

$$S_{i,j-1}^0 + I_{i,j} - \theta_{p,j} Oy_i^{f,p} - Sp_{i,j} = S_{i,j}^0; \quad \forall_j \tag{2}$$

$$\theta_{0.7,j} = \begin{cases} 0.8 & \text{for failure years} \\ 1.0 & \text{for successful years} \end{cases}$$

2. Over-year active storage volume capacity

$$S_{i,j-1}^0 \leqslant Y_i^0; \quad \forall_j \tag{3}$$

3. Within-year storage continuity

$$S_{i,t-1}^w + \beta_{i,t}(Oy_i^{f,p}) - (Oy_{f,p}^{i,t}) = S_{i,t}^w; \quad \forall_t \tag{4}$$

where $\beta_{i,t}$ is the fraction in time t of total annual yield assumed as critical period inflow (usually taken as the ratio of the inflow in period t of the driest year of record to the total annual flow that year).

4. Total reservoir capacity

$$Y_i^0 + S_{i,t-1}^w \leqslant Ya_i; \quad \forall_t \tag{5}$$

5. Proportioning of yields in within-year periods

$$Oy_{f,p}^{i,t} = K_{i,t}(Oy_i^{f,p}); \quad \forall_t \tag{6}$$

where $K_{i,t}$ is the proportion of annual reservoir yield for irrigation to be supplied in time period t.

The 9 year annual flow values taken from Loucks et al. (1981) are 4.0, 3.0, 3.0, 2.0, 1.0, 3.0, 6.0, 8.0, and 6.0. The fourth and fifth years are taken as failure years. A failure fraction of 0.8 is applied to the annual yield during failure years to satisfy the allowable deficit criterion. The $\beta_{i,t}$ values are assumed to be 0.5 for both periods. The values for the factor $K_{i,t}$ are assumed to be 0.6 for the first period and 0.4 for the second period.

The values of annual yields obtained for this 9 year, two-period, single-yield problem from a reservoir capacity of 2.5 after solution are 3.09 during successful years and 2.47 during failure years.

Equivalent Multiple-Yield Formulation

A yield model formulation with two yields, one firm (90% reliable) and the other secondary (70% reliable), is presented here for the same data. An additional constraint is applied to monitor the proportion of annual yields during successful and failure years for accommodating the allowable deficit criterion. The problem is now formulated to find the minimum reservoir capacity to give the same value of annual yield (sum of firm and secondary yields in this case), that is, 3.09. The secondary annual and time (t) period yields with a reliability p are denoted by $Oy_i^{s,p}$ and $Oy_{s,p}^{i,t}$, respectively. The model formulation with two yields for the 9 year, two-period problem shall be as follows:

$$\text{Minimize} \quad Ya_i \tag{7}$$

Subject to

1. Over-year storage continuity

$$S_{i,j-1}^0 + I_{i,j} - Oy_i^{f,p} - \theta_{p,j} Oy_i^{s,p} - Sp_{i,j} = S_{i,j}^0; \quad \forall_j \tag{8}$$

$$\theta_{0.7,j} = \begin{cases} 0 & \text{for failure years} \\ 1 & \text{for successful years} \end{cases}$$

2. Over-year active storage volume capacity

$$S_{i,j-1}^0 \leqslant Y_i^0; \quad \forall_j \tag{9}$$

3. Within-year storage continuity

$$S_{i,t-1}^w + \beta_{i,t}(Oy_i^{f,p} + Oy_i^{s,p}) - (Oy_{f,p}^{i,t} + Oy_{s,p}^{i,t}) = S_{i,t}^w; \quad \forall_t \tag{10}$$

4. Total reservoir capacity

$$Y_i^0 + S_{i,t-1}^w \leqslant Ya_i; \quad \forall_t \tag{11}$$

5. Proportioning of yields in within-year periods

$$Oy_{f,p}^{i,t} + Oy_{s,p}^{i,t} = K_{i,t}(Oy_i^{f,p} + Oy_i^{s,p}); \quad \forall_i \tag{12}$$

6. Constraint for the allowable annual deficit criterion: The incorporation of the allowable deficit in annual yield is achieved by setting the annual firm yield equal to the failure fraction times the sum of the annual firm and secondary yields. (It is to be noted that the value of $\theta_{0.7,j}$ in the over-year storage continuity Eq. (8) being zero, the annual target to be supplied from the reservoir during a failure year shall be governed by the quantity of the firm yield only) and is given by

$$\sum_t Qy_{f,p}^{i,t} = \left[\frac{\theta_{p,j}}{(1-\theta_{p,j})}\right]\left(\sum_t Oy_{s,p}^{i,t}\right); \quad \forall_i \tag{13}$$

The solution for this formulation gives identical results with a reservoir capacity of 2.5 as the single-yield problem. The reliability of the annual yield in the single-yield formulation and the total annual yield (the sum of the firm and secondary yields) in the equivalent multiple-yield formulation is maintained the same as the values of annual yields during an equal number of successful and failure years that are identical.

Multiple-Yield Model for Multiple-Reservoir System

Continuity of Yields among Reservoirs and Incorporation of Allowable Deficit

The single-reservoir model presented in the previous section incorporates an allowable deficit criterion by converting a single-yield problem to a multiple-yield problem while maintaining the desired reliability. Such a conversion can overcome the difficulty in maintaining the continuity of yields among different reservoir sites in a multiple-reservoir model incorporating single-purpose and multipurpose reservoirs.

Consider a multiple-reservoir system having single-purpose irrigation reservoirs, single-purpose hydropower reservoirs, and multipurpose reservoirs with irrigation and hydropower as the two purposes. Using the concept illustrated, a single-purpose irrigation reservoir, which normally is represented by a single-yield problem, can be represented by a two-yield formulation where the irrigation target is the sum of firm and secondary yields, achieving the desired reliability and an allowable deficit criterion. A single-purpose hydropower reservoir can be represented by a two-yield model using the firm and secondary yields for firm and secondary energy generation, respectively. In the case of a multipurpose reservoir, the irrigation target shall be the sum of the firm and secondary yields, whereas the two yields shall be separately available for firm and secondary energy generation. The constraint for the allowable annual irrigation deficit criterion is to be included only for reservoirs having an irrigation component. As every reservoir in the system now has two yields, each having the same reliability throughout the system, there is no difficulty in writing the continuity equations at different reservoir sites in the system. The development of a multiple-yield model for a multiple-reservoir system is presented in the next section.

Model Development

The present multiple-reservoir formulation is aimed at including two yields: one firm and the other secondary. The purposes considered are energy generation (both firm and secondary) and irrigation. It is assumed that the release for energy generation shall be available for irrigation after producing energy. An annual allowable deficit criterion during the failure years is to be incorporated for irrigation. The objective of this model is to maximize the returns from energy generation for known reservoir and hydroplant capacities. Let P denote the set of exceedence probabilities p to be considered. The index i refers to a reservoir site, index j refers to a year, index t refers to a within-year period, and index k refers to a contributing reservoir among the set of m contributing reservoirs upstream of reservoir i. The basic equations in the model are presented below, and the necessary modifications explained in the previous sections are effected while applying the model to a multiple-reservoir system.

Maximize the returns from energy generation as follows:

$$\text{Maximize} \quad \sum_i \left[(B_i^f \; E_i)+(B_i^s \; \overline{E_i})\right] \tag{14}$$

Both B_i^f and B_i^s are returns from annual firm (E_i) and secondary ($\overline{E_i}$) energies, respectively, for reservoir i.

Subject to

1. Over-year storage continuity for year j at reservoir i

$$S_{i,j-1}^0+\left[\sum_{k\in m} \quad Sp_{k,j}\right]+I_{i,j}-Oy_i^{f,p}-\theta_{p,j}Oy_i^{s,p}-E1_{i,j}-Sp_{i,j}$$
$$=S_{i,j}^0; \quad \forall_{i,j} \tag{15}$$

$$\theta_{p,j}=\begin{cases}0 & \text{for failure years}\\ 1 & \text{for successful years}\end{cases}$$

where $E1_{i,j}$=annual evaporation volume loss from reservoir i in year j.

2. Over-year active storage volume capacity for year j at reservoir i

$$S_{i,j-1}^0 \leqslant Y_i^0; \quad \forall_{i,j} \tag{16}$$

3. Within-year storage continuity for reservoir i in time t (regenerated flows are to be added for each i^{th} reservoir having m upstream contributing reservoirs)

$$S_{i,t-1}^w+\beta_{i,t}\left[(Oy_i^{f,p}+Oy_i^{s,p})+\sum_t E1^{i,t}\right]+\sum_{k\in m}\{[\delta_k^f(Oy_{f,p}^{k,t})]$$
$$+[\delta_k^s(Oy_{s,p}^{k,t})]\}-E1^{i,t}-(Oy_{f,p}^{i,t}+Oy_{s,p}^{i,t})=S_{i,t}^w; \quad \forall_{i,t} \tag{17}$$

where $E1^{i,t}$=Evaporation volume loss from reservoir i in period t.

If a reservoir i is affected by the regulation of upstream reservoirs, the within-year yields $Oy_{f,p}^{i,t}$ and $Oy_{s,p}^{i,t}$ are the total yields at that reservoir site in each period t. They include the upstream yields that flow into the reservoir i. The annual yields $Oy_i^{f,p}$ and $Oy_i^{s,p}$ [Eq. (15)] do not include the upstream yields that flow into the reservoir i. The upstream yields are not included in the over-year storage continuity equation at site i, so it is possible to define the within-year inflow distribution of the incremental annual yields $Oy_i^{f,p}$ and $Oy_i^{s,p}$. The within-year inflow distribution of the natural incremental annual yield $(Oy_i^{f,p}+Oy_i^{s,p})$ defined by $\beta_{i,t}$s in Eq. (17) is not likely to be the same as the controlled within-year outflow distributions of the yields $Oy_{f,p}^{k,t}$ and $Oy_{s,p}^{k,t}$ from the upstream reservoirs (Loucks et al. 1981).

4. Total active reservoir storage capacity for reservoir i

$$Y_i^0+S_{i,t-1}^w \leqslant Ya_i; \quad \forall_{i,t} \tag{18}$$

5. Definition of estimated evaporation losses in year j for reservoir i

$$E1_{i,j}=E0_i+\left[S_{i,j-1}^0+\sum_t\left(\frac{S_{i,t-1}^w+S_{i,t}^w}{2}\right)\gamma_{i,t}\right]E1_i^r; \quad \forall_{i,j} \tag{19}$$

where $E1_i^r$=average annual evaporation volume loss rate per unit of active storage volume for reservoir i; $E0_i$=average annual fixed evaporation volume loss due to dead storage for reservoir i; and $\gamma_{i,t}$=fraction of annual evaporation volume loss from reservoir i in period t.

6. Definition of estimated evaporation losses in time t (assuming that the initial over-year storage volume $S_{i,cr}^0$ in the critical year is zero) for reservoir i

$$E1^{i,t}=\gamma_{i,t}E0_i+\left(S_{i,cr}^0+\frac{S_{i,t-1}^w+S_{i,t}^w}{2}\right)\gamma_{i,t}E1_i^r; \quad \forall_{i,t} \tag{20}$$

$S_{i,cr}^0$=Initial over-year storage volume in critical year

7. Continuity of annual yields at each reservoir site (regenerated flows are to be added for each i^{th} reservoir having m upstream contributing reservoirs)

For firm yield

$$\sum_t OY_{f,p}^{i,t}=Oy_i^{f,p}+\sum_{k\in m}\left[\delta_k^f\sum_t (Oy_{f,p}^{k,t})\right]; \quad \forall_i \tag{21}$$

For secondary yield

$$\sum_t Oy_{s,p}^{i,t}=Oy_i^{s,p}+\sum_{k\in m}\left[\delta_k^s\sum_t (Oy_{s,p}^{k,t})\right]; \quad \forall_i \tag{22}$$

where δ_k^f and δ_k^s are fractions of firm and secondary yields respectively coming as regenerated flow from upstream reservoir k.

8. Irrigation target constraint for reservoir i in time t

$$Oy_{f,p}^{i,t}+Oy_{s,p}^{i,t}=K_{i,t}\left((Oy_i^{f,p}+Oy_i^{s,p})+\sum_{k\in m}\left\{\left[\delta_k^f\sum_t (Oy_{f,p}^{k,t})\right]\right.\right.$$
$$\left.\left.+\left[\delta_k^s\sum_t (Oy_{s,p}^{k,t})\right]\right\}\right); \quad \forall_{i,t} \tag{23}$$

9. Constraint for allowable annual deficit criterion (for reservoirs having irrigation component)

$$\sum_t Oy_{f,p}^{i,t} \geqslant \left[\frac{\theta_{p,j}}{(1-\theta_{p,j})}\right]\left(\sum_t \quad Oy_{s,p}^{i,t}\right);$$
$$\forall_{i,i\in\text{reservoirs having irrigation component}} \tag{24}$$

This constraint is made greater than or equal to in the multireservoir formulation so as to allow the model to have flexibility in deriving the benefits of energy generation from single purpose hydropower and multipurpose reservoirs.

10. Firm energy generation for reservoir i in time t

$$E_{i,t}=(\text{CF},e_i,Ha_{i,t})Oy_{f,p}^{i,t} \quad \forall_{i,t} \tag{25}$$

where CF=conversion factor for computation of hydroelectric energy; e_i=hydropower plant efficiency for reservoir i; and $Ha_{i,t}$=productive storage head for reservoir i in period t.

11. Secondary energy generation for reservoir i in time t

$$\overline{E_{i,t}}=(\text{CF},e_i,Ha_{i,t})Oy_{s,p}^{i,t}; \quad \forall_{i,t} \tag{26}$$

12. Plant capacity limitations for reservoir i in time t

$$E_{i,t}+\overline{E_{i,t}} \leqslant (\alpha_{i,t},h_{i,t})H_i; \quad \forall_{i,t} \tag{27}$$

where $\alpha_{i,t}$=hydropower plant factor for reservoir i in pe-

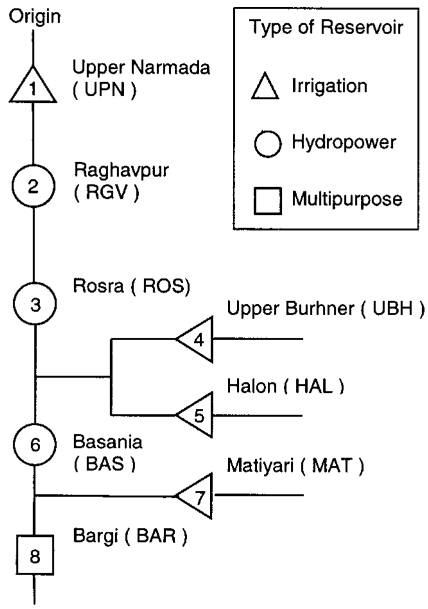

Fig. 1. Line diagram of eight reservoirs in upper basin of Narmada River

riod t; H_i=hydropower plant capacity for reservoir i; and $h_{i,t}$=number of hours for generation of energy for reservoir i in period t.

13. Firm energy target constraint for reservoir i in time t

$$E_{i,t}=\eta_{i,t}E_i; \quad \forall_{i,t} \tag{28}$$

where $\eta_{i,t}$=percentage fraction of annual firm energy target for reservoir i in period t.

14. Annual surplus energy generation at reservoir i

$$\sum_t \overline{E_{i,t}}=\overline{E_i}; \quad \forall_i \tag{29}$$

Application of Model

This example illustrates the application of a yield model to a multiple-reservoir system consisting of single-purpose and multipurpose reservoirs. The yield model above is applied to a system of eight major reservoirs in the Narmada River basin system in central India. The line diagram in Fig. 1 shows the system. Though some of these reservoirs are proposed, it is presumed for analysis that all reservoirs and hydroplants are existing within their stipulated Master Plan capacities. Out of these reservoirs four are single-purpose irrigation, three are single-purpose hydropower, and one multipurpose. The model includes two purposes: hydroelectric energy generation (both firm and secondary) and irrigation.

The necessary data is obtained from the Master Plan (Government 1972) and Waikar (1998). The flow-record period is of 22 years. Annual firm reservoir yield with a reliability of $p=[22/(22+1)]=0.96$ (no failure year) and secondary reservoir yield with a reliability of $p=[(22-5)/(22+1)]=0.74$ (five failure years) are considered in the model to achieve the reliabilities of 74% for irrigation, 96% for firm energy, and 74% for secondary energy. The five failure years were selected by visual inspection of the annual flow values and confirmed after making a few trials with the model. A maximum of a 20% deficit in the irrigation target is to be allowed during failure years. The mean monthly flows at reservoir sites are presented in Fig. 2(a), and the standard deviation of monthly flows at reservoir sites is shown in Fig. 2(b). Two of these reservoirs, Basania and Bargi, have to provide drinking water supply with a small demand, which has been deducted from the inflows at these sites. The flows at the reservoirs at Upper Narmada, Upper Burhner, Halon, and Matiyari shall be reduced due to the upstream use by the proposed medium, minor, and pumping schemes. The estimated values of this upstream use less the regenerated flows are deducted from the inflows at the respective reservoir sites.

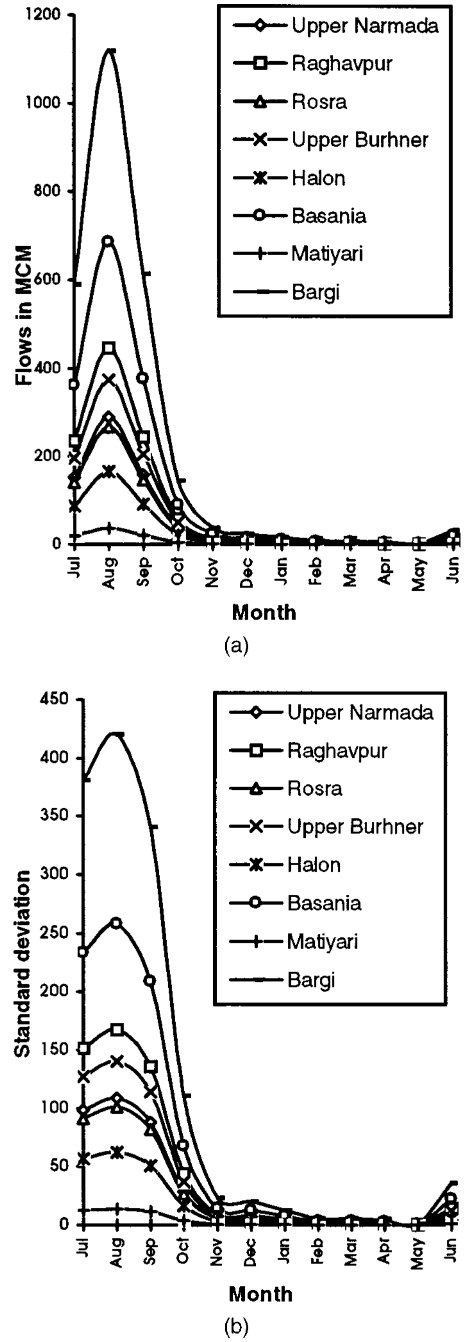

Fig. 2. (a) Mean monthly flows at reservoir sites; (b) standard deviation of monthly flows at reservoir sites

The model considers three within-year time periods, each of four months duration. The water year starts from the month of July. The parameter β_t, which reflects the relative proportion of

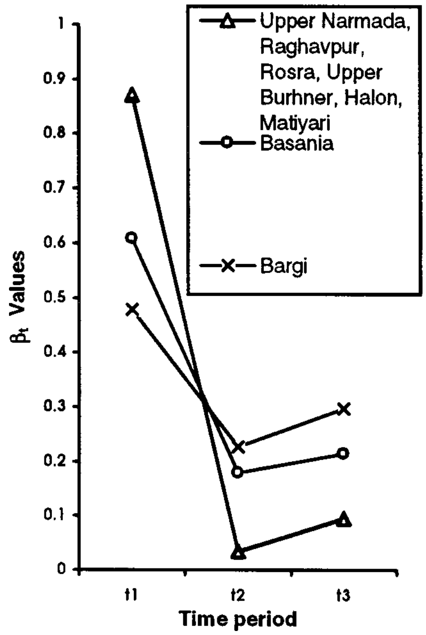

Fig. 3. Values of β_t at reservoir sites

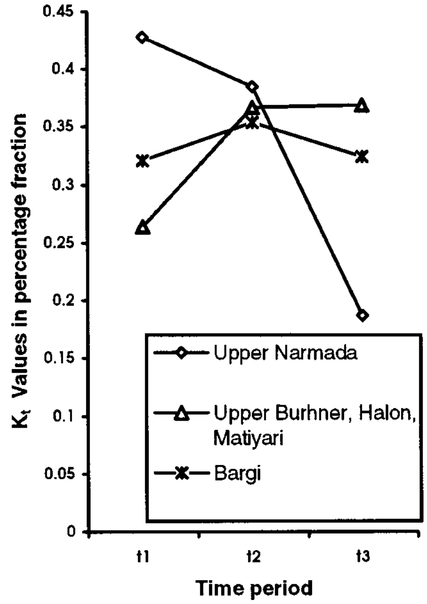

Fig. 5. Values of K_t at reservoir sites

the critical year's inflow that is likely to occur in period t, is taken as the ratio of the inflow in period t of the driest year on record to the total inflow in that year. These β_t values at reservoir sites are presented in Fig. 3. Storage-area curves (linearized above dead storage) and storage-elevation curves are used for computation of parameters in evaporation equations and substitutions in energy equations, respectively. The values of parameter γ_t (the fraction of the annual evaporation volume loss that occurs in period t) at reservoir sites are presented in Fig. 4, and the percentage fractions of irrigation target for the three within-year periods at reservoir sites are presented in Fig. 5. Equal distribution among periods is assumed for the firm energy, whereas no restriction over the distribution of secondary energy among the within-year periods is imposed. The regenerated flow percentages are assumed to be 10% for irrigation and 100% for hydropower release. The energy calculations are based on a plant factor of 60%. The values of returns from firm and secondary energy are assumed to be Rs. 2,000.0 and Rs. 700.0 per megawatt hour (MWH). Irrigation is given priority over generation of hydropower as per the National Water Policy in India. The objective of the model is to find the maximum energy generation targets for given levels of irrigation targets. The monetary values of returns from energy generation are used only to depict the relative significance of firm and secondary energies.

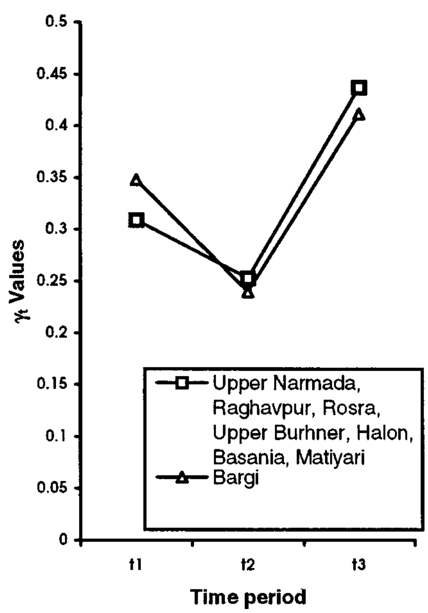

Fig. 4. Values of γ_t at reservoir sites

Discussion of Results

The four irrigation reservoirs at Upper Narmada, Upper Burhner, Halon, and Matiyari were first analyzed individually to find their maximum irrigation targets for a reliability of 74% and an allowable deficit of 20%. The annual irrigation targets of these four reservoirs were found to be 186, 396, 84, and 33 MCM (million cubic meters), respectively. The multiple-reservoir model was then solved for four cases. In the multiple-reservoir formulation, these individual irrigation yield model targets were set as lower bounds for these reservoirs to achieve their individual maximum targets in Case I, and the results were obtained. The annual system irrigation target and firm power target were found to be 4,880 MCM and 39.2 MW respectively.

Next the annual targets of the four irrigation reservoirs upstream of reservoir Bargi were reduced to examine the effect on the individual firm energy targets: the targets of reservoir Bargi as well as the total system targets. Accordingly, three more cases were examined by reducing the irrigation targets of the four irrigation reservoirs by 10, 20, and 30% for Cases II, III, and IV, respectively, and the model was solved. The results of these four cases are presented in Table 1. The productive heads were substituted externally into the model, starting with average values, and were verified after obtaining the solution. The process was iterated with refined values of heads until the storages obtained were equivalent to the values of heads and the annual irrigation and firm power values stabilized. The model could have been solved

Table 1. Yield Model Results of Four Cases

Item		UPN	RGV	ROS	UBH	HAL	BAS	MAT	Total	BAR	Grand total
Live capacity (MCM)		253	252	497	592	158	1799	51.12	3602.12	3,180	6,782.12
Plant capacity (MW)		—	20	35	—	—	60	—	115	90	205
Over-year reservoir capacity (MCM)	Case I	168.4	70.9	283.7	343.5	104.7	1586	28.5	2,248.9	2,406	4,654.9
	Case II	176.9	60.1	275.9	368.3	110.0	1565	30.8	3,587.0	2,424	5,011.0
	Case III	185.4	51.5	276.8	393.2	115.4	1570	33.0	2,625.3	2,403	5,028.3
	Case IV	193.8	43.5	277.1	418.0	120.7	1555	35.3	2,643.4	2,401	5,044.4
Annual irrigation target (MCM)	Case I	185.9	—	—	396.0	83.65	—	32.67	698.22	4,182	4,880.22
	Case II	167.3	—	—	356.4	75.28	—	29.40	628.38	4,245	4,873.38
	Case III	148.7	—	—	316.8	66.92	—	26.14	558.56	4,307	4,865.56
	Case IV	130.2	—	—	277.2	58.55	—	22.87	488.82	4,369	4,857.82
Annual firm power target (MW)	Case I	—	3.22	7.20	—	—	9.44	—	19.86	19.37	39.23
	Case II	—	3.40	7.20	—	—	9.60	—	20.20	20.30	40.50
	Case III	—	3.59	7.20	—	—	9.95	—	20.79	20.92	41.71
	Case IV	—	3.74	7.39	—	—	10.20	—	21.33	21.71	43.04
Annual irrigation target (MCM)	Master Plan target project+ (diversion)	280+(**216**)	—	—	150+(**395**)	175	—	71.54	676.54+(**611**)	3530+(**2,060**)	4,206.54+(**2,671**)

Note: (1) Annual firm power target is obtained by converting the firm energy from solution of the yield model; (2) values in brackets (in bold type) are the proposed annual irrigation diversion targets in the Master Plan.

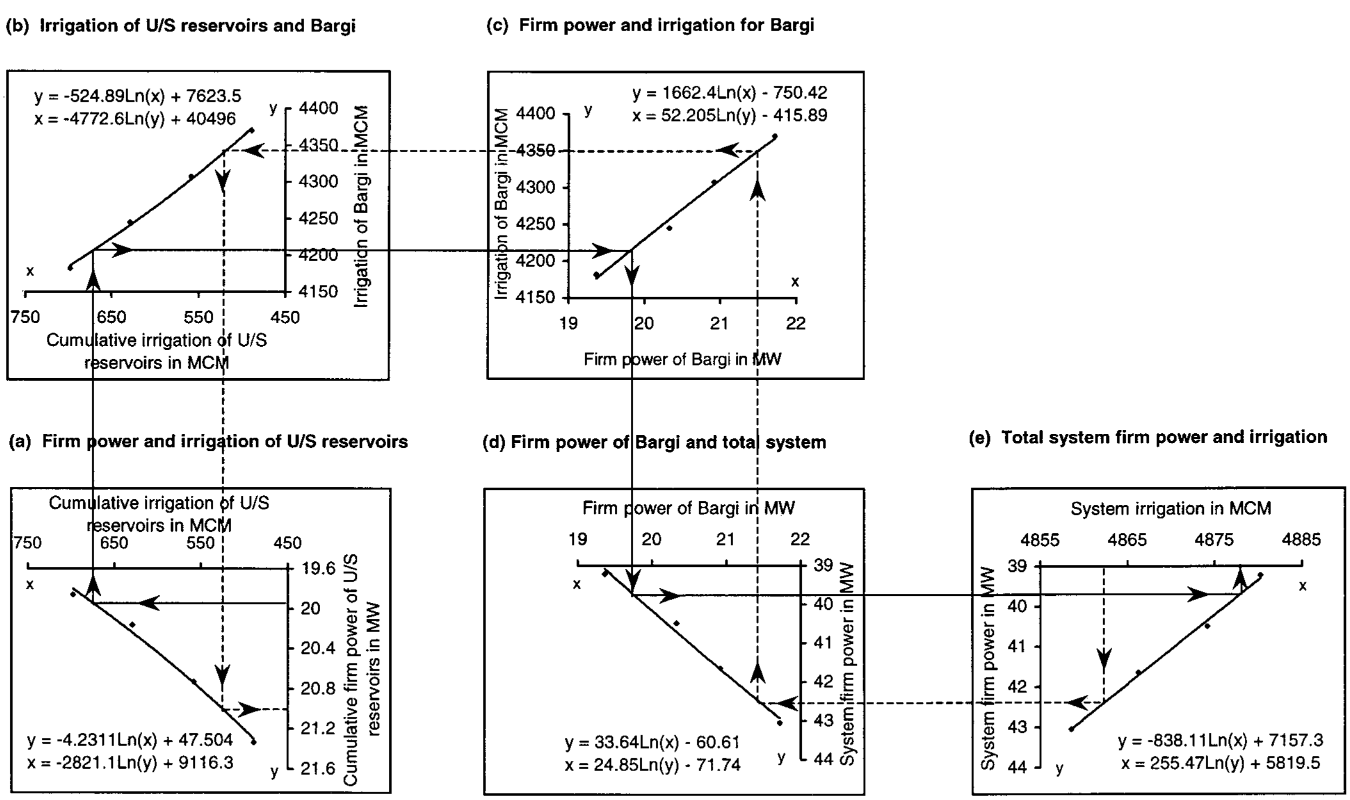

Fig. 6. Trade-off between firm power and irrigation of system with reference to Bargi

for 12 (monthly) or more time periods in a water year. However, three periods were considered to reduce the model size, as the main purpose was to demonstrate the feasibility and applicability of the approach. Also this may be sufficient given that inflow occurs totally with a well-defined season [dominant monsoon hydrology; see Figs. 2(a and b)]. The LP yield model had 715 constraints and 700 variables. Three seconds of CPU time was required on a Pentium computer (Celeron 433 Mhz processor) for one solution (excluding the time for model preparation and loading) by the LINDO package.

The results presented in Table 1 show that, as the targets of the four upstream (U/S) irrigation reservoirs are reduced, the firm power increases for the purely hydropower reservoirs, namely, Raghavpur, Rosra, and Basania, which are located on the main river. This trade-off between annual firm power and irrigation for reservoirs upstream of reservoir Bargi is presented in Fig. 6(a). Other trade-offs between irrigation of upstream reservoirs and irrigation of Bargi, between firm power with irrigation for reservoir Bargi, and between firm power for Bargi and the total system are shown in Figs. 6(b–d), respectively. There is an increase in both irrigation and firm power of reservoir Bargi with a reduction in the irrigation target of upstream reservoirs. The trade-off between total system firm power and irrigation is presented in Fig. 6(e).

These five trade-offs can be used to derive the information about the relative variations in the various system targets during the planning stages. For this purpose two paths, are shown in Fig. 6, one indicated by the firm line and the other by the dashed line. The path indicated by the firm line can be used if the decisions are to be taken with reference to the cumulative irrigation or firm power targets of the reservoirs upstream of reservoir Bargi. Similarly the path indicated by the dashed line can be used to successively derive the desired information if the decisions are to be taken with reference to the total system targets. The graphical presentation in Fig. 6 indicates the mode of extraction of information. However, for precise calculations, the equations given for the trade-off curves shall have to be used.

The 75% annual dependable flow at reservoir Bargi is 3,318 MCM after accounting for 10% regenerated flow by the medium, minor, and pumping schemes. The total system irrigation target (excluding the proposed annual irrigation diversion targets; refer to Table 1) as stipulated in the Master Plan for development of the water resources of the Narmada in Madhya Pradesh (Government 1972) and the Narmada Control Authority (1994), is 4,206.54 MCM, and as per the yield model results, the system irrigation target ranges between, 4,858 and 4,880 MCM for the four cases solved. As regards the firm hydropower potential of the system under consideration, the estimate as per the Master Plan is 63.31 MW annually, whereas the system's annual firm power ranges between 39.23 and 43.04 MW for the four cases analyzed. The estimated total effect of carry-over storages in assessing the total net utilizable 75% annual dependable flow for planning the development of the entire Narmada River basin as assumed in the report of the Narmada Water Disputes Tribunal (Government 1978), is 3,700 MCM. The 75% dependable carry-over storage that can be provided by the system of these eight reservoirs ranges between 961 and 1292 MCM for the four cases considered, which is 26 to 35% of 3,700 MCM.

The irrigation targets stipulated in the Master Plan for development of the water resources of the Narmada in Madhya Pradesh (Government 1972) are worked out considering the 75% annual dependable flow available at each project site. Where 75% annual dependable flow is not adequate, 60% annual dependable flow has been considered. It is stated finally that the effective reservoir yield at 75% dependability due to carryover storage will be taken into account. However, there is no consideration for the percentage of annual yield to be made available from the reservoir during failure years. The yield model results are based on the full-length historical record of annual inflows implying the reliability of reservoir yield rather than the use of one year flow of a specified dependability. Moreover, as the percentage of annual yield to be made available from the reservoir is specified by using the failure fraction, the extent of failure during failure years can be monitored.

The estimate of annual reservoir yield without considering the failure fraction shall always be on the conservative side as the extent of failure cannot be controlled. This can at times lead to very severe failures during some of the failure years having low flows, making the reservoir system more vulnerable. Hashimoto et al. (1982) provided clear illustrations of the concept of vulnerability, which is a measure of the significance (extent) of yield failure that supplements the more common reliability criteria by providing a more complete picture of risk in reservoir performance. The vulnerability criterion used by Moy et al. (1986) is the magnitude of the largest deficit during the period of operation. The failure fraction employed in the yield model can be one way to represent the vulnerability of a reservoir system. Thus the yield model offers some distinct advantages over the deterministic linear programming screening models using one-year flow of specified dependability by providing superior reservoir yield estimates.

Summary and Conclusions

This study conducted from an academic research perspective is an effort to improve the reservoir yield model and to apply the multiyield model to a multireservoir system. The proposed yield model can achieve the desired reliabilities for irrigation and energy generation and incorporates an allowable annual deficit in the irrigation target. The yield model applied to a multiple-reservoir system demonstrated its use in assessing the irrigation and hydropower potential of the system and their relative effects. The results of the four cases are used to generate alternative scenarios to assist the planners in decision making.

The proposed yield model offers a flexible modeling structure with a straightforward translation of the concept of annual yield reliability and allowable deficit while maintaining independent identities of the firm and secondary reservoir yields for the representation of different water uses. The failure fraction can be effectively employed to monitor the extent of yield failure and to represent the vulnerability of a reservoir system. The present application of the model is for monsoon hydrology. This improves the precision of the approximation of splitting the over-year and within-year storage based on an assumption that the inflows and required releases are just in balance, so that the reservoir neither fills nor empties during the modeled critical year. This is expected in the critical year that generally occurs at the end of a drawdown period. The estimation of evaporation volume losses is also relatively more accurate [Eq. (20)].

Modification of the yield model to incorporate an allowable deficit in an annual irrigation target required that the annual firm reservoir yield was a "safe" yield without any failure year. This puts a theoretical restriction on exercising complete control over the reliability of water uses that may be represented by the annual firm (safe) reservoir yield, as the annual reliability of a safe reservoir yield $[p=n/(n+1)]$ is always governed by the length n

(in years) of the historical flow record. However, this does not put any limitation on the practical utility of the multiple-yield model developed in this study, as the attempt is usually made to achieve the maximum reliabilities for water uses such as domestic and industrial water supply and firm hydropower generation, which may be represented by the annual firm (safe) yield in a water resources system.

The proposed modification in the yield model for a multiple-reservoir system resolves the difficulty in achieving the desired annual reliabilities for different water uses while maintaining the continuity in annual yields with different reliabilities among reservoirs. This is in addition to the incorporation of an allowable deficit in the annual target for the water use having a desired reliability less than the maximum reliability (irrigation, in this study). A common value of failure fraction is employed in the present study for the entire system. The modification made in the yield model conceptually provides a complete flexibility in specifying separate values of failure fraction in Eq. (13) at each reservoir site ($\theta_{p,i,j}$) in the system, or to monitor the value of failure fraction (between zero and one) by considering it as a decision variable. Hence, the proposed yield model can act as a powerful tool in monitoring the allowable deficits in the annual targets of water uses such as irrigation to enable a more meaningful and logical represention of the important concept of vulnerability of reservoir systems.

The work left undone is a critical evaluation of this yield model with detailed simulation. The anticipated difficulties in such a system simulation are incorporation of an allowable deficit in the annual irrigation target and achieving the desired reliabilities for the firm and secondary hydropower generations and irrigation, especially in the case of the multipurpose reservoirs. The results obtained in this study can assist in easing this tedious and time-consuming task by providing superior initial estimates.

The yield model given here is sufficiently accurate and could be applied to similar multiple-reservoir systems. Though the model is applied to a system with known reservoir capacities, it can also be used for examining the design alternatives of a proposed system. As the desired annual reliabilities for yields can be prespecified for different purposes by consideration of an allowable deficit in annual yield in a multiple-reservoir system, this yield model can act as a better screening tool in planning by providing outputs that can be very useful in improving the efficiency and accuracy of detailed analysis methods such as simulation.

Acknowledgments

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