CONDITIONAL DISTRIBUTIONS OF IDEAL RESERVOIR STORAGE VARIABLES

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ABSTRACT: Storage capacity, yield, and reliability of a reservoir are random variables. Conditional distributions of these variables are studied by Monte Carlo simulation for operating periods up to 100 years. It is shown that these distributions are affected by the length of the period of operation due to the assumption of an initially full reservoir. Results of the study will aid in the reliability based reservoir design by clarifying the connection among the storage variables. The relationship between the steady-state reliability and the probability of failure-free operation over an N-year planning period is also investigated, and it is shown that the effect of the initial storage can be neglected only when the ratio of regulation is low.

INTRODUCTION

Storage reservoirs are important components of water resource systems. In the design of reservoirs, the risk of failure should be taken into account as a reservoir of a given size can provide a certain yield only with a certain probability because of the random nature of inflows.

Engineers usually design a reservoir to regulate the observed series of streamflows, in which case the risk of failure cannot be determined. The population characteristics of flows must be considered when a probabilistic design is required. In this case storage capacity, yield, and risk are dependent random variables. The joint and/or conditional probability distributions of these variables should be known to make meaningful decisions. The objective of this study is to investigate the conditional distributions of these variables for an ideal reservoir operated with the standard policy when inflows are independent normal variables. Results are expected to clarify the relationship among the storage variables.

In earlier studies, Pegram (1980) and Pegram et al. (1980) defined the basic variables for storage calculations and introduced their nondimensional forms. Klemes (1981) presented a state-of-the art report. Bayazit (1982), Vogel and Stedinger (1987), and Bayazit and Bulu (1991) obtained capacity-yield-risk relationships for ideal reservoirs. Most of these studies make an assumption that a reservoir that is full in the beginning of the operation period.

The paper starts with the definition of the variables in the study. The steady-state probability of success and the probability of success for a finite operating period are introduced as two different but interrelated concepts, both of which have been used in reservoir design studies. The steady-state probability of success is obtained as a function of reservoir capacity and yield by simulation, and it is compared with earlier results. The conditional probability distributions of the storage variables are then obtained for finite operating periods up to 100 years by simulation for certain combinations of the values of these variables. The simulation results are presented in the form of graphs and tables. It is shown on an example how the information obtained in this study can be employed in reservoir design.

The final part of the paper concerns the investigation of the relationship between the steady-state and finite period probabilities of success. It is shown that an earlier result that has been widely used is valid only for small yields because of the effect of the initial storage.

BASIC CONCEPTS

An ideal reservoir is a concept that is frequently used in the preliminary stages of reservoir design for over-year regulation. All losses from the reservoir are neglected or subtracted from the inflow. The reservoir is operated with the standard policy, such that the release is equal to the chosen target (yield) when the available water (storage content in the beginning of the year plus the inflow during the year) is sufficient, called the "success" or "regular state"; otherwise, the reservoir is emptied, with the occurrence of the failure state.

For any reservoir the storage variables K (storage capacity), Y (yield), and p (reliability defined as the annual probability of success) are interrelated. With the increase of the operating period N, the reliability will approach a steady-state value p_s for given K and Y. For a certain inflow model (probability distribution and autocorrelation structure) and a chosen initial condition (initial reservoir content), the relationship between the storage variables can be expressed as (Klemes 1981)

$$p_s = fn(K, \mu_x, \sigma_x, Y) \tag{1}$$

where μ_x and σ_x = mean and standard deviation of annual inflows, respectively.

The above equation can be made nondimensional as follows (Pegram et al. 1980):

$$p_s = fn(K_\sigma, C_{vx}, Y_\sigma) \tag{2}$$

where $K_\sigma = K/\sigma_x$; $C_{vx} = \sigma_x/\mu_x$; and $Y_\sigma = Y/\sigma_x$. Introducing the parameter $m = (\mu_x - Y)/\sigma_x$ (Pegram et al. 1980), (2) can be rewritten as

$$p_s = fn(K_\sigma, C_{vx}, m) \tag{3}$$

The probability of success p in any year equals the probability of available water exceeding the required yield $P[V + x \geq Y]$, where V is the reservoir storage at the beginning of the year, and x is the inflow during the year. For normally distributed inflows, this probability can be written as

$$p = P[V + x \geq Y] = P[x \geq Y - V] = P\left[\frac{x - \mu_x}{\sigma_x} \geq \frac{Y - \mu_x}{\sigma_x} - \frac{V}{\sigma_x}\right]$$

$$= P\left[z \geq -m - \frac{V}{\sigma_x}\right] = 1 - \Phi\left(-m - \frac{V}{\sigma_x}\right) \tag{4}$$

where z = standard normal variate; and Φ = standard normal cumulative distribution function. It is seen that for normal inflows it suffices to consider the variable $C_{vx} = \sigma_x/\mu_x$ in the

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TABLE 1. Steady-State Reliability p_s as Function of K_σ and m

K_σ (1)	m 0.0 (2)	0.2 (3)	0.4 (4)	0.6 (5)	0.8 (6)	1.0 (7)
0.25	0.549	0.633	0.711	0.780	0.839	0.885
	0.549	0.635	0.714	0.783	0.841	0.887
	0.550	0.635	0.713	0.783	0.843	0.887
0.50	0.595	0.684	0.763	0.830	0.882	0.921
	0.596	0.686	0.766	0.832	0.884	0.923
	0.597	0.687	0.766	0.832	0.884	0.922
1.00	0.675	0.770	0.846	0.903	0.942	0.967
	0.675	0.773	0.851	0.907	0.944	0.968
	0.675	0.772	0.848	0.905	0.943	0.967
2.00	0.776	0.874	0.937	0.972	0.988	0.996
	0.775	0.879	0.943	0.976	0.990	0.996
	0.776	0.875	0.938	0.972	0.988	0.995
4.00	0.863	0.954	0.988	0.998	~0	~0
	0.857	0.955	0.990	0.998	—	—
	0.863	0.954	0.988	0.997	—	—
8.00	0.923	0.992	~0	~0	~0	~0
	0.917	0.992	—	—	—	—
	0.923	0.991	—	—	—	—
16.00	0.959	~0	~0	~0	~0	~0
	0.955	—	—	—	—	—
	0.959	—	—	—	—	—

Note: The first entry in columns 2–7 is simulation result of present study, middle entry is from Buchberger and Maidment (1989), and lower entry is from Pegram (1980).

parameter m. Thus, (3) can be simplified in the case of normal inflows as

$$p_s = fn(K_\sigma, m) \tag{5}$$

Table 1 shows the relationship between K_σ and p_s for m = 0(0.2)1 obtained by Monte Carlo simulation with independent normally distributed flows (each simulation run was 10,000,000 years long). The agreement with the diffusion approximation results of Buchberger and Maidment (1989), and particularly with the finite-difference equation solutions of Pegram (1980), is very good.

DISTRIBUTIONS OF STORAGE VARIABLES FOR CERTAIN OPERATING PERIOD

For a finite operating period of N years, the storage variables K_σ, m, and p_N (reliability over N years, defined as the probability that no failure occurs in N years or that the reservoir supplies the required yield throughout the operating period) are random variables. For given values of two of these variables, the conditional probability distribution of the remaining variable can be obtained. Thus, the cumulative distribution functions $F(K_\sigma|m, p_N)$, $F(m|K_\sigma, p_N)$, and $F(p_N|K_\sigma, m)$ are defined for certain period N.

Limited information is available concerning these distributions. Most of the studies have been carried out for the case of perfect reliability ($p_N = 1$), also called deficit analysis. Vogel (1985) developed expressions for the distribution $F(K_\sigma|m, p_N = 1)$ in the case of AR(1) normal inflows. Bayazit (1982) and Bayazit and Bulu (1991) investigated the same function for normal and lognormal autoregressive [AR(1)] and autoregressive moving average [ARMA(1,1)] inflows, N = 25 and 50 years. Vogel and Stedinger (1987) derived expressions for $F(K_\sigma|C_{vx}, m, p_N = 1)$, $N \leq 100$ years for the lognormal AR(1) model. Phien (1993) generalized the study to gamma inflows. A three-parameter lognormal distribution was found to fit quite well to the function $F(K_\sigma|m, p_N = 1)$ for all inflow models investigated in these studies.

Recently, Pretto et al. (1997) obtained certain quantiles of the distribution $F(K_\mu|C_{vx}, m, p_N)$ for p_N = 0.90 and 0.95, N = 10–5,000 years, in the case of AR(1) normal and lognormal inflows, where $K_\mu = K/\mu_x$. They found that for $m \leq 0.5$, the mean and upper quantiles of the storage capacity had a downward bias for small N, and an upward bias for larger N before the convergence to a steady level, whereas the median and lower quantiles of K_μ always showed a downward bias.

STATISTICAL PROPERTIES OF STORAGE VARIABLES FOR SHORT OPERATING PERIODS

Most reservoirs have design periods less than N = 100 years. In this section the relationship among the storage variables K_σ, m, and p_N for such short operating periods is analyzed. For this purpose a Monte Carlo simulation study is carried out for certain combinations of the values of these variables to determine their conditional distributions.

In all of the simulations, inflows were generated by an independent normal model as the earlier studies showed that the behavior of the variables was similar for all inflow models. N was varied in the range N = 25(25)100. The parameter m was equal to 0.25 to characterize the reservoirs with medium resilience, just above the value of 0.2, below which the two-state Markov model was found to be inadequate to represent the structure of failure sequences (Vogel and Bolognese 1995); values of m smaller than 0.25 are avoided in practice. The reliability was chosen as p_N = 0.95–1.0 because a risk of failure over N years of about 5% is usually tolerated. The reservoir was assumed to be full initially, as usual in the deficit analysis. In each simulation the inflow sequence of N years was routed twice through the reservoir, and the reliability was estimated only for the second cycle, with the expectation that the influence of initial conditions would be removed by concatenating the inflow sequence with itself (Pretto et al. 1997). Three conditional probability distribution functions were investigated for the selected values of the parameters, as described below. For each set of parameters the number of simulations was 50,000.

Distribution $F(p_N|K_\sigma, m)$

This function is useful in practice for determining the distribution of the reliability of a reservoir of given capacity supplying a certain yield.

A reservoir of capacity K_σ = 3 was operated for a yield corresponding to m = 0.25 over N years, N = 25(25)100. The steady-state reliability for this case is p_s = 0.94 from Table 1. The distribution functions $F(p_N|K_\sigma = 3, m = 0.25)$, determined by simulation, are shown in Fig. 1, and the quantiles of p_N are given in Table 2.

All of the distribution curves intersect at about p_N = 0.92, below which F values decrease as N increases; whereas above p_N = 0.92, F values increase with N. Although the mean of p_N is always equal to the steady-state reliability, the quantiles vary with N. $q_{0.10}$ increases from 0.84 for N = 25 to 0.89 for N = 100. $q_{0.25}$ remains practically constant, whereas the median and upper quantiles decrease as N increases. These results are in agreement with those of Pretto et al. (1997) for m below about 0.5. The minimum value of the reliability in 50,000 simulations increases considerably with N, from 0.36 for N = 25 to 0.67 for N = 100.

In the study, the probability of perfect reliability $P[p_N = 1|K_\sigma = 3, m = 0.25]$ was also determined. This probability drops from 0.502 for N = 25 to 0.063 for N = 100 years.

Distribution $F(m|K_\sigma, p_N)$

This function is important for determining the distribution of the yield that a reservoir of a given capacity can supply with a certain reliability over N years.

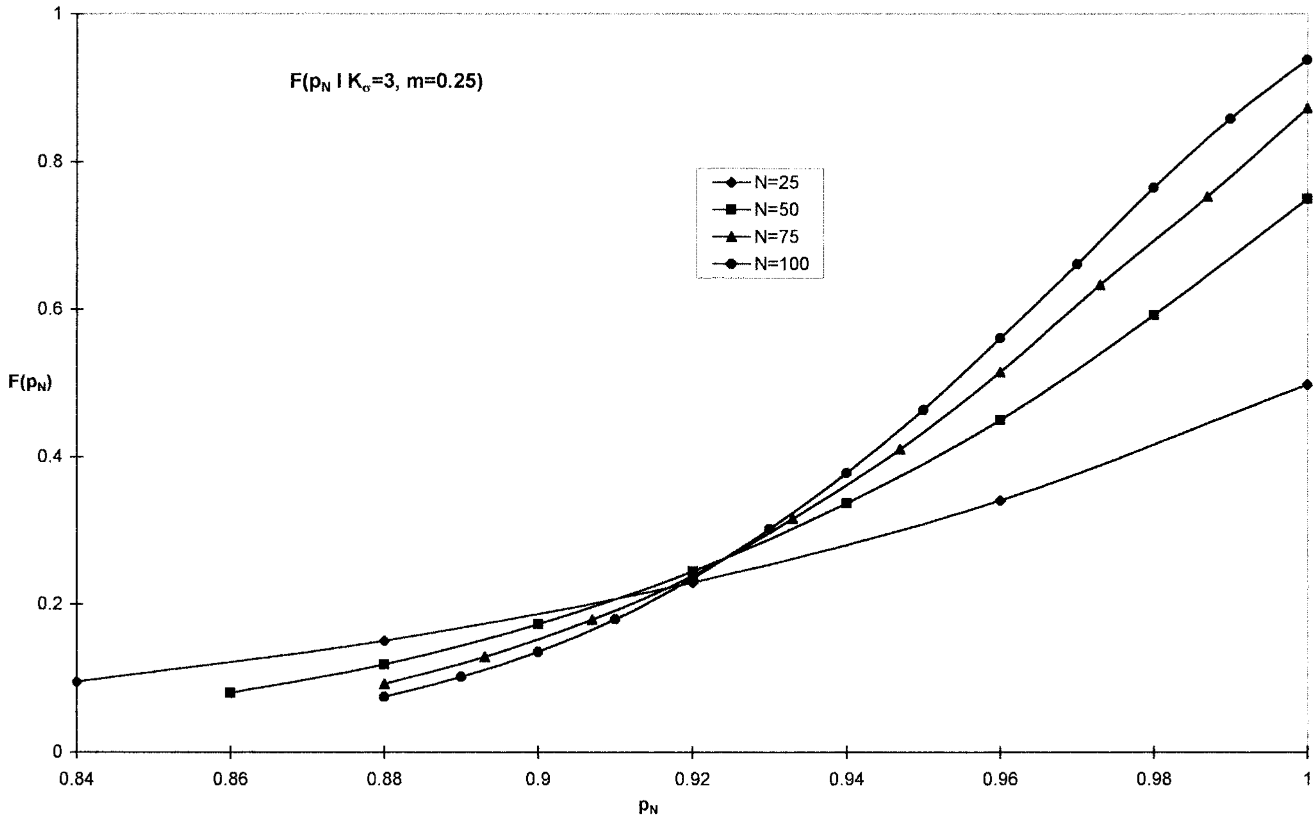

FIG. 1. Conditional Distribution of p_N for K_σ = 3 and m = 0.25

TABLE 2. Statistical Characteristics of Distribution $F(p_N|K_\sigma = 3, m = 0.25)$

Parameter (1)	N 25 (2)	50 (3)	75 (4)	100 (5)
Mean	0.94	0.94	0.94	0.94
$q_{0.90}$	1.00	1.00	1.00	0.995
$q_{0.75}$	1.00	1.00	0.99	0.98
$q_{0.50}$	1.00	0.97	0.96	0.95
$q_{0.25}$	0.93	0.92	0.92	0.92
$q_{0.10}$	0.84	0.87	0.88	0.89
Minimum	0.36	0.52	0.55	0.67

The same reservoir of capacity K_σ = 3 was operated for N years with yields corresponding to various values of m in the range m = 0–0.9, corresponding to over-year regulation. In each simulation the reliability p_N was evaluated. Then the ratio of the runs with $p_N \geq 0.96$ and $p_N = 1.0$ were determined. Thus, the distribution $F(m|K_\sigma = 3, p_N)$ was obtained for p_N = 0.96 and p_N = 1, N = 25(25)100 (Figs. 2 and 3). Simulation results are summarized in Table 3 for p_N = 0.96, and in Table 4 for p_n = 1.

For the reliability p_N = 0.96, all of the distribution curves intersect at about m = 0.5, above which F increases slightly with N, whereas below m = 0.5 the probability F decreases with the increase of N. The lower quantiles and median of m increase with N. This means that as the design period increases, the probability that the yield that can be supplied with a reliability of p_N = 0.96 over an N-year period exceeds a given value, decreases for values of m below about 0.5. Higher quantiles remain almost constant or decrease slowly as N increases.

For perfect reliability (p_N = 1), all of the quantiles increase with the operating period N. This increase is more pronounced than in the case of p_N = 0.96. As an example, F(0.5) = 0.80 for N = 25 years, and F(0.5) = 0.42 for N = 100 years. Thus, the probability that a yield corresponding to m = 0.5 or a larger yield can be guaranteed over a 25-year period is 0.80, whereas it is only 0.42 for a 100-year period.

Distribution $F(K_\sigma|m, p_N)$

This function gives the distribution of the capacity of a reservoir that supplies a given yield with a certain reliability over N years.

The distribution of the reservoir capacity K_σ for perfect reliability over N years (p_N = 1) was determined by application of the sequent peak algorithm to two cycles of inflow sequences of length N = 25(25)100 years. The distribution function $F(K_\sigma|m = 0.25, p_N = 1)$ is plotted in Fig. 4 and its quantiles are given in Table 5.

All of the quantiles of K_σ are seen to increase significantly as N increases. For example, the median is K_σ = 3.00 for N = 25, and K_σ = 5.12 for N = 100. The mean is always higher than the median, indicating that the conditional distribution of the reservoir capacity is positively skewed. In fact, it is known that K_σ corresponding to p_N = 1 is lognormally distributed with a skewness of 1.625 (Bayazit and Bulu 1991).

Comments

Using the conditional distribution functions derived above, statements of the following kind can be made, which are basically equivalent.

1. The probability that a reservoir of size K_σ = 3 can supply a yield corresponding to m = 0.25 with a reliability p_N = 1 (or higher, which is of course not meaningful in this case) over N = 25 years is 0.50 (Fig. 1).
2. The probability that a reservoir of size K_σ = 3 can supply with perfect reliability (p_N = 1) a yield corresponding to m = 0.25 (or a larger yield) over N = 25 years is 0.50 (Fig. 3).

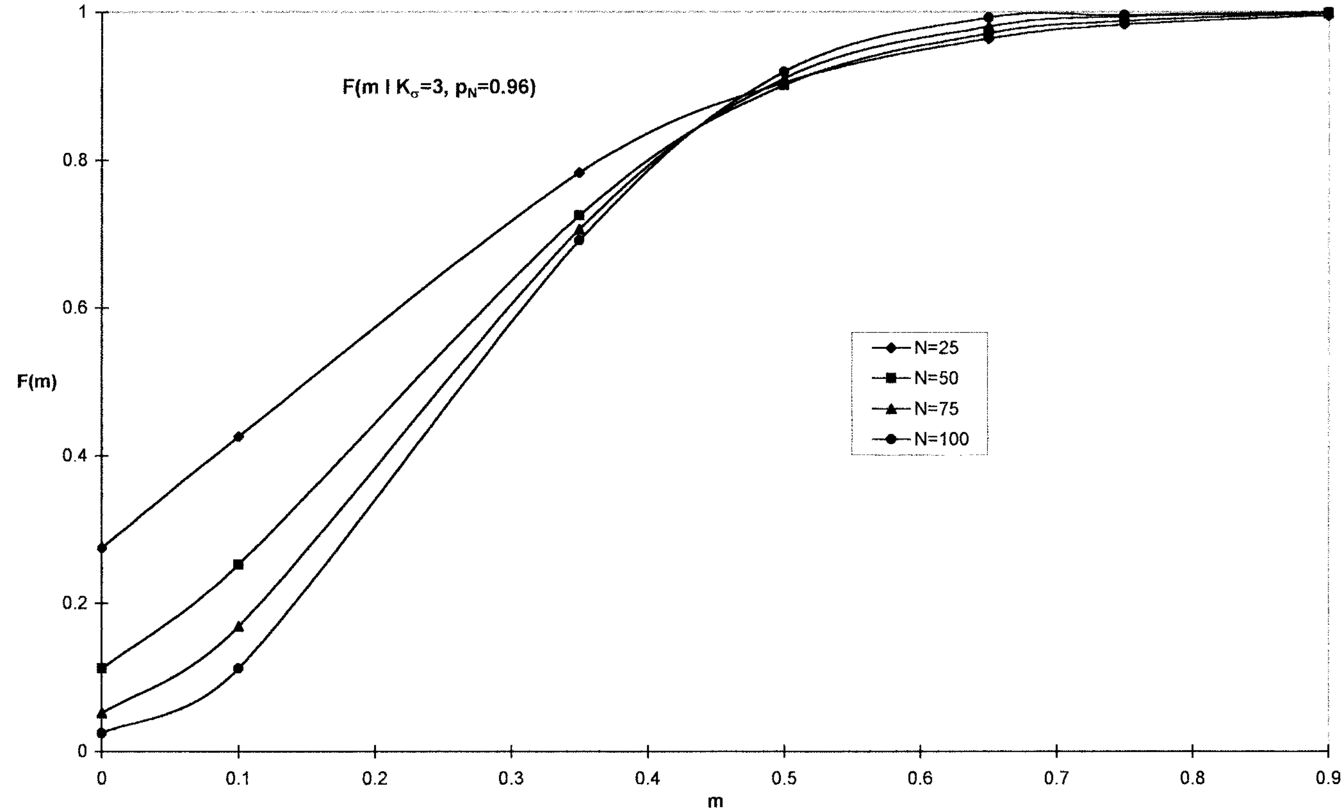

FIG. 2. Conditional Distribution of m for K_σ = 3 and p_N = 0.96

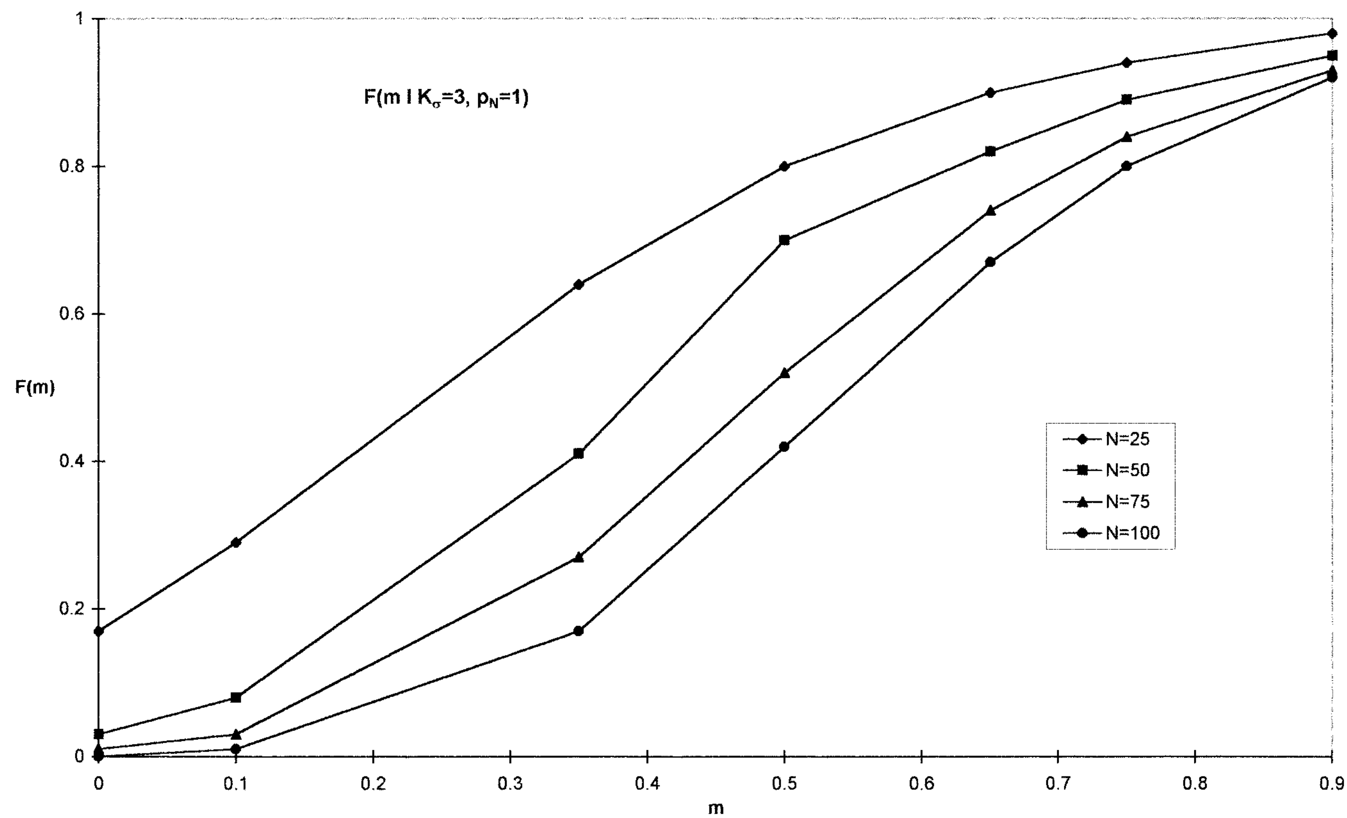

FIG. 3. Conditional Distribution of m for K_σ = 3 and p_N = 1

3. The probability that a reservoir of size K_σ = 3 or smaller will be needed to supply a yield corresponding to m = 0.25 with perfect reliability (p_N = 1) over N = 25 years is 0.50 (Fig. 4).

Note that the probability in the above statements would be equal to 0.25, 0.13, and 0.06 over N = 50, 75, and 100 years, respectively, indicating the effect of the reservoir being initially full.

TABLE 3. Statistical Characteristics of Distribution $F(m|K_\sigma = 3, p_N = 0.96)$

Parameter (1)	N 25 (2)	50 (3)	75 (4)	100 (5)
$q_{0.90}$	0.50	0.50	0.50	0.48
$q_{0.75}$	0.33	0.36	0.38	0.38
$q_{0.50}$	0.15	0.22	0.25	0.27
$q_{0.25}$	0.00	0.10	0.12	0.14
$q_{0.10}$	0.00	0.00	0.05	0.09

TABLE 4. Statistical Characteristics of Distribution $F(m|K_\sigma = 3, p_N = 1)$

Parameter (1)	N 25 (2)	50 (3)	75 (4)	100 (5)
$q_{0.90}$	0.64	0.77	0.83	0.90
$q_{0.75}$	0.45	0.58	0.65	0.71
$q_{0.50}$	0.25	0.40	0.49	0.54
$q_{0.25}$	0.07	0.25	0.34	0.41
$q_{0.10}$	0.00	0.12	0.22	0.30

EXAMPLE

A reservoir is to be designed for a period of $N = 50$ years. The annual inflows are normally distributed with $\mu_x = 1{,}050$ and $\sigma_x = 200$, there is no serial correlation. The desired annual yield is $Y = 1{,}000$. The reservoir storage capacity is chosen as $K = 600$ (all figures are in 10^6 m^3). The reservoir is assumed to be full in the beginning of the operation period.

Nondimensional parameters have the values $K_\sigma = K/\sigma = 600/200 = 3$, $m = (\mu_x - Y)/\sigma = (1{,}050-1{,}000)/200 = 0.25$.

1. The steady-state reliability is $p_s = 0.94$ from Table 1, implying that the annual probability of success will approach 0.94 as the effect of initial condition vanishes. But this value is approached asymptotically, and it would be interesting to know the conditions in the first 50 years.
2. What is the value of the reliability in the design period p_{50}? p_{50} is a random variable. From Table 2 its mean is 0.94, it will be higher than 0.92 with a probability of 0.75, higher than 0.97 with a probability of 0.50, and equal to 1.0 with a probability of 0.25. There is a 25% chance that the reservoir will never be emptied during the period of 50 years, for a yield of 1,000.
3. What is the yield that can be supplied with perfect reliability ($p_{50} = 1$) over 50 years? From Table 4, m will be less than 0.25 with a probability of 0.25, less than 0.40 with a probability of 0.50, and less than 0.58 with a probability of 0.75. There is a 25% chance that the supplied yield will exceed 1,000, a 50% chance that it will exceed 970, and a 75% chance that it will exceed 934 throughout the design period.
4. Let it be assumed that a risk of 0.04 is allowed in 50 years, corresponding to $p_{50} = 0.96$. Now we can read from Table 3 that m will be less than 0.10 with a probability of 0.25, less than 0.22 with a probability of 0.50, and less than 0.36 with a probability of 0.75. Therefore, the yields that can be provided with 25, 50, and 75% chances are raised to 1,030, 1,006 and 978, respectively, as a result of accepting a 4% risk of failure in the design period.
5. If perfect reliability is required, what is the storage capacity that will provide a yield of 1,000? This capacity is a random variable. From Table 5, its mean is $K_\sigma = 4.53$ ($K = 906$), it will be less than $K_\sigma = 3.00$ ($K = 600$) with a probability of 0.25, less than $K_\sigma = 3.99$ ($K = 798$) with a probability of 0.50, and less than $K_\sigma = 5.44$ ($K = 1{,}088$) with a probability of 0.75.

The tables and figures provided in this paper are only for the case of $K_\sigma = 3$ and $m = 0.25$. It is possible to prepare similar graphs for other combinations of the variables, other inflow models, and different initial conditions, but this would

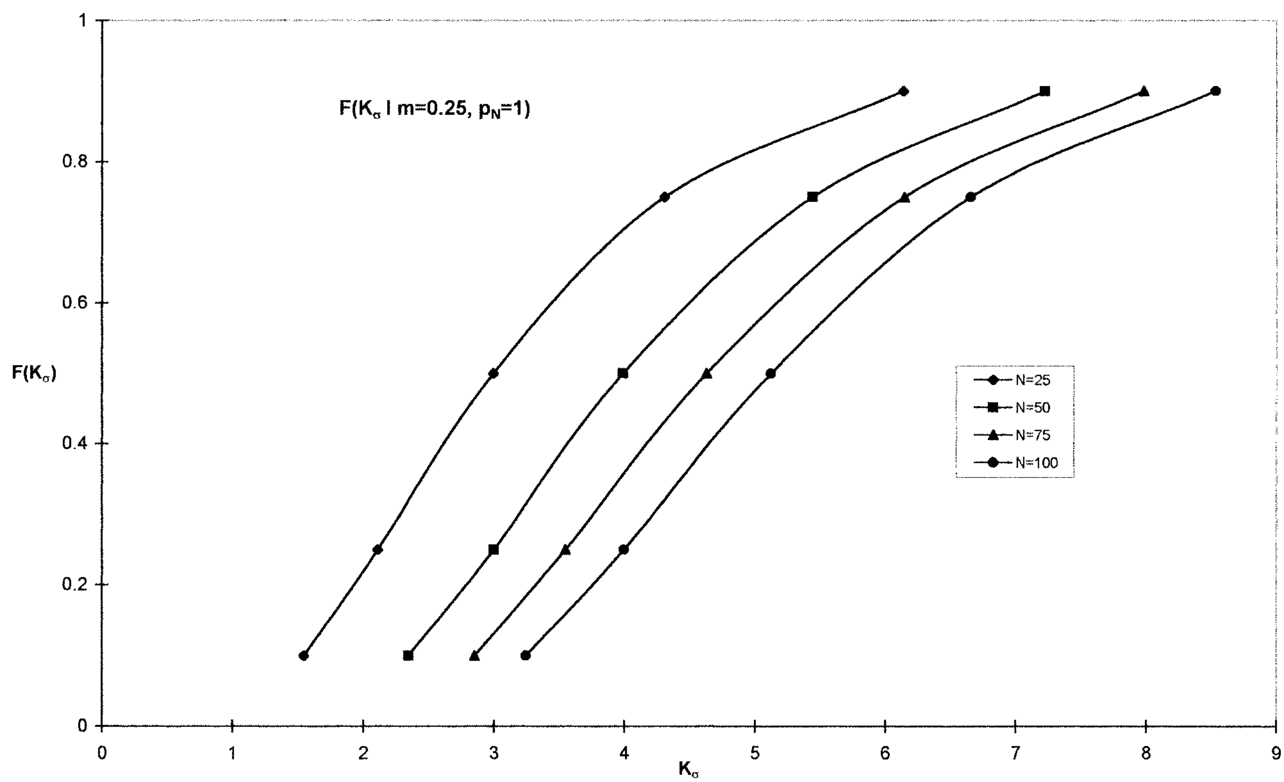

FIG. 4. Conditional Distribution of K_σ for $m = 0.25$ and $p_N = 1$

TABLE 5. Statistical Characteristics of Distribution $F(K_\sigma|m = 0.25, p_N = 1)$

Parameter (1)	N 25 (2)	50 (3)	75 (4)	100 (5)
Mean	3.59	4.53	5.14	5.60
$q_{0.90}$	6.14	7.23	7.98	8.53
$q_{0.75}$	4.31	5.44	6.14	6.66
$q_{0.50}$	3.00	3.99	4.63	5.12
$q_{0.25}$	2.11	3.00	3.55	4.00
$q_{0.10}$	1.55	2.35	2.85	3.25
Maximum	29.46	38.23	43.33	34.33

require too much effort and space. The purpose here is to clarify the relationship among the variables p_N, K_σ, and m, and to illustrate how such information could be used in the preliminary design of reservoirs.

RELATIONSHIP BETWEEN p_s AND $P[p_N = 1]$

Considering a reservoir of capacity K supplying a yield corresponding to a given value of m for a period of N years, the probability of no-failure operation throughout the operating period $P[p_N = 1]$, in the case of the initially full condition, is

$$P[p_N = 1] = P[S_1 \cap S_2 \cap \cdots \cap S_N|V_0 = K] \quad (6)$$

where S_i denotes regular (successful) operation in year i; and V_0 = initial reservoir content. Vogel (1987) argued that the ''steady-state'' probability of failure-free operation over an N-year planning period equals the probability of normal operation in the first year p_s, multiplied by the probability that subsequent years remain free of failures

$$P[p_N = 1] = p_s(1 - f)^{N-1} \quad (7)$$

where f = probability that a failure year follows a regular year. Comparing (7) with (6), it is seen that the effect of the initial condition is neglected and the probabilities of success are expressed in terms of steady-state probabilities.

Vogel and Bolognese (1995) used the probabilities $P[p_N = 1]$ obtained by Vogel (1985) for normal inflows, and by Vogel and Stedinger (1987) for lognormal inflows to compute the probabilities p_s by (7), and then compared them with their Monte Carlo-simulation results and the results of Pegram (1980) and Buchberger and Maidment (1989). They found that (7) was useful in converting the no-failure reliability over an N-year planning period to steady-state reliability p_s for values of m in excess of 0.2. They argued that the two-state Markov model used in deriving the equation could not provide an adequate description of reservoir failures in the case of low resilience ($m < 0.2$) because such systems may take several years to refill once empty.

The results of the present study are used to check the above equation for $m = 0.25$, in which case a satisfactory agreement would be expected according to the findings of Vogel and Bolognese (1995). Table 6 shows the simulation results $P[p_N = 1]$ for $K_\sigma = 3$, n = 25(25)100, compared with the corresponding values obtained from (7) with $p_s = 0.94$ read from Table 1, and $f = 0.60\ (1 - p_s)/p_s = 0.037$, where 0.60 is the probability that a regular year follows a regular year for independent normal inflows in the case of $m = 0.25$ $(1 - \Phi(-0.25) = 0.60)$. It is seen that the estimates of (7) are substantially lower than the simulation results for all values of N, in contradiction to the findings of Vogel and Bolognese (1995). The reason why the probabilities of no-failure operation over an N-year period estimated by (7) do not agree with the simulation results is that the effect of the initial condition (full reservoir) leads to much higher reliabilities in the simulations. Routing the flow sequence twice through the reservoir and estimating the reliability only for the second cycle is apparently not sufficient to remove the influence of initial conditions.

TABLE 6. Simulation Results for $P[p_N = 1]$ Compared with Eq. (7)

(1)	N 25 (2)	50 (3)	75 (4)	100 (5)
$P[p_N = 1]$ (simulation)	0.502	0.250	0.128	0.063
$P[p_N = 1]$ [Eq. (7)]	0.381	0.149	0.058	0.023

Vogel and Bolognese (1995) compared the steady-state reliability p_s, estimated by (7), with the simulation results, and found good agreement for $m > 0.2$. The reason for this is that the equation is not sensitive to differences in $P[p_N = 1]$ when solved for p_s. In fact, when the simulation results of the present study are used to estimate p_s using (7), a value of $p_s = 0.955$ is found with $P[p_N = 1]$ of $N = 25$ and 50, and $p_s = 0.956$ with $P[p_N = 1]$ of $N = 75$ and 100, which are not much different from the true reliability $p_s = 0.94$.

It can be concluded that (7) should not be used to estimate the probability of no-failure operation over an N-year period, even for values of m in excess of 0.2. Only for much higher values of m close to 1.0 does the agreement become satisfactory because the reservoir will soon be filled whatever the initial condition is.

CONCLUSIONS

For finite operating periods, the variables relevant to reservoir design (storage capacity, yield, and reliability) have random character. Assuming that the reservoir is initially full, the behavior of their conditional probability distributions is studied by simulation for certain values of the variables that are meaningful in reservoir design. It is seen that these distributions vary with the length of the period of operation because the reservoir is assumed to be full in the beginning. The results shed light on the probabilistic behavior of the storage variables and can aid in making decisions in reservoir design problems.

The equation given by Vogel (1987) for the relationship between steady-state and finite period probabilities is shown to give satisfactory results only when the ratio of regulation is low ($m \sim 1$ or higher).

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

C_{vx} = coefficient of variation of annual inflows, σ_x/μ_x;
F = probability distribution function;
K = reservoir storage capacity;
K_μ = K/μ_x;
K_σ = K/σ_x;
m = $(\mu_x - Y)/\sigma_x$;
N = reservoir operating period;
P = probability;
p = annual reliability;
p_N = reliability over N years;
p_s = steady-state reliability;
q_i = ith quantile;
S_i = success in year i;
V = storage in beginning of year;
V_0 = initial reservoir storage;
x = annual inflow;
Y = yield;
Y_σ = Y/σ_x;
z = normal variate;
μ_x = mean of annual inflows;
σ_x = standard deviation of annual inflows; and
Φ = standard normal distribution function.