WATER RESOURCES BULLETIN
VOL. 23, NO. 6 AMERICAN WATER RESOURCES ASSOCIATION DECEMBER 1987

COMPARISON OF RESERVOIR LINEAR OPERATION RULES USING LINEAR AND DYNAMIC PROGRAMMING[1]

Nageshwar Rao Bhaskar and E. Earl Whitlatch[2]

ABSTRACT: Mathematical optimization techniques are used to study the operation and design of a single, multi-purpose reservoir system. Optimal monthly release policies are derived for Hoover Reservoir, located in Central Ohio, using chance-constrained linear programming and dynamic programming-regression methodologies. Important characteristics of the former approach are derived, discussed, and graphically illustrated using Hoover Reservoir as a case example. Simulation procedures are used to examine and compare the overall performance of the optimal monthly reservoir release policies derived under the two approaches. Results indicate that, for the mean detention time and the corresponding safe yield target water supply release under existing design of Hoover Reservoir, the dynamic programming policies produce lower average annual losses (as defined by a two-sided quadratic loss function) while achieving at least as high reliability levels when compared to policies derived under the chance-constrained linear programming method. In making this comparison, the reservoir release policies, although not identical, are assumed to be linear. This restricted form of the release policy is necessary to make the chance-constrained programming method mathematically tractable.
(KEY TERMS: optimization; reservoir operation; multi-purpose reservoir; simulation; chance-constrained linear programming; dynamic programming.)

INTRODUCTION

Mathematical optimization algorithms such as linear and dynamic programming have been successfully applied in the study of single, multi-purpose reservoir systems. Young (1966) used the latter approach to derive annual operating policies for a single reservoir. The release policies were obtained by regressing optimal releases on important variables in the reservoir system. A similar approach was adopted by the authors (Bhaskar and Whitlatch, 1978) to derive reservoir release policies on a monthly time scale. ReVelle, *et al.* (1969), introduced the chance-constrained linear programming method for deriving optimal release policies for a reservoir system operating under probabilistic constraints. This technique incorporates the degree of system failure explicitly, but is generally mathematically tractable only when used in conjunction with a linear decision rule release policy (ReVelle, *et al.*, 1969; Loucks, 1970; Loucks and Dorfman, 1975). A comprehensive review of the state-of-the-art of mathematical models used in studying reservoir operation, including the above two methods, is given in Stedinger, *et al.* (1984), and Yeh (1985).

Studies to examine the relative merits of optimization techniques currently used in reservoir management and design studies are, however, lacking. In an effort to meet this need, this paper compares monthly release policies for a single multi-purpose reservoir system using chance constrained linear programming with similar but not identical policies obtained by the dynamic programming-regression methodology. Hoover Reservoir, located on Big Walnut Creek near Columbus, Ohio, is selected as a case study. Based on synthetically generated inflows, the reservoir is simulated using release policies derived by the two methods to determine system performance.

CASE STUDY DESCRIPTION

Hoover Reservoir is currently used to supply a draft of 75 MGD to partially meet the water needs for the City of Columbus, Ohio. A reservoir storage of 58,154 acre-feet is allocated for this purpose. Dead storage is 2,188 acre-feet, and a storage equal to 25,780 acre-feet is reserved for flood control. The top of water supply pool represents a mean detention time (defined as the ratio of active reservoir capacity to mean annual inflow) of 0.44 years indicating a strictly within-year storage. The reservoir has been in operation since September 1954.

Statistical analysis of monthly historical flows, prior to and after the construction of the reservoir, shows that the inflows are log-normally distributed (Bhaskar, 1978). Important statistics based on natural inflow data from 1939-1954 are given in Table 1. An examination of the lag-one correlation coefficient of inflows indicated that, except for the high flow months of February and March and low-flow months of September and October, these coefficients were significant at the 5 percent level of significance. Furthermore,

[1]Paper No. 86071 of the *Water Resources Bulletin.* **Discussions are open until August 1, 1988.**
[2]Respectively, Associate Professor, Department of Civil Engineering, Speed Scientific School, University of Louisville, Louisville, Kentucky 40292; and Associate Professor, Department of Civil Engineering, Ohio State University, Columbus, Ohio 43210.

a detailed analysis of higher lagged correlation coefficients showed that for most months current inflow was not significantly correlated with higher order lags than the first (Bhaskar, 1978). Consequently, the use of Markov first order model to synthesize streamflows is justified. A Markov first order model as proposed by Fiering (1971) is used to synthetically generate monthly inflows based on statistics in Table 1. This model generates the current month inflow using a deterministic component, incorporating the mean monthly inflow of the previous month (to reflect persistence), and a random component (to reflect noise) whose probability distribution depends on probability distribution of monthly inflow.

An estimate of safe-yield (once in 50-year failure) can be obtained through use of the sequent peak algorithm or by simulation of the standard policy at selected target levels. Synthetically generated inflows are applied in either method. Both approaches suggest that the gross safe-yield (including losses) under the existing Hoover Reservoir design is close to 75 MGD (Bhaskar, 1978). The standard policy for reservoir operation can be stated as follows. First if there is not enough water to meet the target demand, release all water from active storage. Second, if there is more than enough water, release enough to meet the target demand, unless there is more water than can be stored, in which case the excess is also released.

MATHEMATICAL DEVELOPMENT

Chance-Constrained Linear Programming

As mentioned earlier, the chance-constrained linear programming approach is mathematically convenient when used in conjunction with a linear release policy. A general form of such a policy for any month t may be written as:

$$X_t = S_{t-1} + \lambda_i R_t - b_i \tag{1}$$

where:

- t = time period in months over the decision period N ($t = 1,2,....,N$);
- i = month ($i = 1,2,....,12$);
- X_t = release in period t;
- S_{t-1} = storage at the end of period $t-1$;
- R_t = inflow in period t;
- b_i = decision constant for month i;
- λ_i = parameter for month i ($0 \leqslant \lambda_i \leqslant 1$); and
- N = decision period in months.

Parameters indexed by i are cyclical over a year and are associated with a particular month, while variables indexed by t are defined every month over the decision period, N. The two indices are related by the expression:

$$i = t (\text{mod } 12) \tag{2}$$

ReVelle and Gundelach (1975) proposed a release rule similar to Equation (1) but incorporating more past period inflows.

Substitution of Equation (1) in the continuity equation, $S_t = S_{t-1} + R_t - X_t$, yields useful forms for defining reservoir storage:

$$S_t = (1 - \lambda_i)R_t + b_i \qquad t = 1,2,\ldots,N \tag{3}$$

or

$$S_{t-1} = (1 - \lambda_{i-1})R_{t-1} + b_{i-1} \qquad t = 1,2,\ldots,N \tag{4}$$

TABLE 1. Statistics of Historical Flows (sample size = 16).

Month	Mean (cfs)	Standard Deviation (cfs)	Coefficient of Variation	Skewness	Lag-One Correlation Coefficient
January	361.426	386.729	1.070	1.053	0.498
February	378.112	210.353	0.556	−0.331	0.351
March	400.069	235.231	0.588	1.027	0.410
April	325.681	185.343	0.569	−0.022	0.380
May	160.931	136.260	0.847	1.463	0.032
June	201.823	206.680	1.024	1.278	0.717
July	94.685	97.337	1.028	1.082	0.644
August	45.921	85.178	1.855	3.660	0.479
September	21.970	39.771	1.810	3.084	0.152
October	11.119	10.301	0.927	0.753	0.166
November	71.539	68.217	0.954	0.404	0.518
December	193.459	209.353	1.082	0.932	0.815

Combining Equations (1) and (4) gives an alternate form of the release policy involving inflows only:

$$X_t = \lambda_i R_t + (1 - \lambda_{i-1})R_{t-1} + b_{i-1} - b_i \qquad t = 1,2,\ldots,N \qquad (5)$$

Optimal release policies derived in this study are expressed in the form of Equation (5) above. For $\lambda_i = 0$, the release policy is similar to the one proposed by ReVelle (1969):

$$X_t = R_{t-1} + b_{i-1} - b_i \qquad t = 1,2,\ldots,N \qquad (6)$$

When $\lambda_i = 1$, the release policy is similar to the one proposed by Loucks (1970):

$$X_t = R_t + b_{i-1} - b_i \qquad t = 1,2,\ldots,N \qquad (7)$$

For λ_i between 0 and 1, the release policy takes the general form:

$$X_t = B_1 R_t + B_2 R_{t-1} + B_0 \qquad t = 1,2,\ldots,N \qquad (8)$$

where the coefficients B_1 and B_2 depend on the value of the λ_i chosen and B_0 is the intercept. The form of the optimal release policies stated in Equation (8) can be derived using dynamic programming as demonstrated in this paper.

The mathematical program for obtaining optimal release policies for a reservoir may be written as:

$$\text{Minimize } C \qquad (9a)$$

subject to:

$$\text{Prob}\,(S_t \leqslant C - v_i) \geqslant \alpha_1 \qquad t = 1,2,\ldots,N \qquad (9b)$$

$$\text{Prob}\,(S_t \geqslant m_i) \geqslant \alpha_2 \qquad t = 1,2,\ldots,N \qquad (9c)$$

$$\text{Prob}\,(X_t \geqslant q_i) \geqslant \alpha_3 \qquad t = 1,2,\ldots,N \qquad (9d)$$

$$\text{Prob}\,(X_t \leqslant f_i) \geqslant \alpha_4 \qquad t = 1,2,\ldots,N \qquad (9e)$$

with

$$X_t, S_t \geqslant 0 \qquad t = 1,2,\ldots,N \qquad (9f)$$

where:

- C = reservoir capacity;
- v_i = flood-control storage in month i;
- m_i = minimum storage in month i;
- q_i = minimum release in month i;
- f_i = maximum release in month i; and
- $\alpha_1, \alpha_2, \alpha_3$, and α_4 = probabilities of satisfying the constraints.

The first two constraints provide reservoir storage for flood-control and recreational purposes with reliabilities α_1 and α_2, respectively. The third constraint assures a minimum release of q_i in month i with reliability α_3, while the last constraint restricts the release from the reservoir below the maximum release f_i with reliability, α_4.

If the probability distribution of inflow, R_t, in any month is assumed to remain unchanged over each year, the number of constraints in each set of Equation (9) will be reduced to 12. Furthermore, the inflow probability distribution, R_t, can be replaced by the corresponding distribution, R_i, in month i, where indices t and i are related as stated in Equation (2). Incorporating Equations (3) through (5), the deterministic equivalent of the chance-constrained program formulation, Equation (9), becomes:

$$\text{Minimize } C \qquad (10a)$$

subject to:

$$C - b_i \geqslant (1 - \lambda_i) r_i^{\alpha_1} + v_i \qquad i = 1,2,\ldots,12 \qquad (10b)$$

$$b_i \geqslant m_i - (1 - \lambda_i) r_i^{(1-\alpha_2)} \qquad i = 1,2,\ldots,12 \qquad (10c)$$

$$b_{i-1} - b_i \geqslant q_i - z_i^{(1-\alpha_3)} \qquad i = 2,3,\ldots,12 \qquad (10d)$$

$$b_{12} - b_1 \geqslant q_i - z_1^{(1-\alpha_3)} \qquad i = 1$$

$$b_{i-1} - b_i \leqslant f_i - z_i^{\alpha_4} \qquad i = 2,3,\ldots,12 \qquad (10e)$$

$$b_{12} - b_i \leqslant f_i - z_1^{\alpha_4} \qquad i = 1$$

with

$$C \geqslant 0;\ b_i \text{ unrestricted} \qquad i = 1,2,\ldots,12 \qquad (10f)$$

where:

- r_i^{α} = inflow corresponding to the cumulative probability of not being exceeded α percent of the time (obtained from the cumulative probability distribution of inflows, R_i, in month i); and
- z_i^{α} = value of random variable Z (as defined in Equation 11 below), corresponding to the cumulative

probability of not being exceeded α percent of the time.

The transformed random variable Z_i can be expressed as:

$$Z_i = \lambda_i R_i + (1 - \lambda_{i-1})R_{i-1} \qquad i = 2,3,\ldots,12 \qquad (11a)$$

$$Z_i = \lambda_i R_i + (1 - \lambda_{12})R_{12} \qquad i = 1 \qquad (11b)$$

An examination of the above equation shows that for $\lambda_i = 1$ for all months, the cumulative probability distribution Z_i is identical to the corresponding cumulative probability distribution of the inflows, R_i, in that month. Alternatively, for $\lambda_i = 0$ for all months, the cumulative probability distribution of Z_i corresponds to the cumulative probability distribution of the inflows in the previous month, R_{i-1}.

In this study, the chance-constrained program developed in Equation (10) is solved by setting the λ_i ($i = 1,2,\ldots,12$) to their extreme values. For λ_i equal to zero the solution corresponds to the model suggested by ReVelle, *et al.* (1969), while a value of unity yields the model proposed by Loucks (1970). As illustrated later in this paper, a more general form of the release policy can be derived using dynamic programming for λ_i values between 0 and 1. Furthermore, the reservoir capacity, C, is incoprorated into the objective function in order to evaluate its relationship to the reliability levels α_1, α_2, α_3, and α_4 of the constraints, as will be illustrated later.

Dynamic Programming-Regression Method

Optimal releases from a single reservoir to meet a target demand, T, may be obtained by solving the following dynamic programming recursive relationship:

$$f_n(S_n) = \min_{X_n} \left[L_n(X_n) + f_{n-1}(S_{n-1})\right] \qquad (n = 1,2,\ldots,N) \qquad (12a)$$

with

$$S_n + R_n - S_{max} \leq X_n \leq S_n + R_n - S_{min} \qquad (12b)$$

$$S_n = S_{n-1} + R_n - X_n \qquad (12c)$$

where:

n = stage number and is equal to time period, t;

$f_n(S_n)$ = total optimal return or loss at stage n;

$L_n(X_n)$ = loss associated with release X_n at stage n;

S_n = storage in the reservoir at the beginning of month n;

X_n = release at stage n;

R_n = inflow at stage n;

S_{min} = minimum storage (or dead storage);

S_f = flood control storage;

S_{max} = maximum storage in the reservoir below the flood pool and is equal to $(C - S_f)$; where C is the reservoir capacity; and

N = number of stages in the dynamic program.

In the above formulation, each stage n corresponds to each month t over the decision period, N. Also, a two-sided quadratic loss function, as defined below, is used for the loss function:

$$L_n(X_n) = (X_n - T_n)^2 \qquad (n = 1,2,\ldots,N) \qquad (13)$$

where T_n is the target release at stage n. In the present study, this is assumed to be at a constant level in each month.

Synthetically generated flow sequences based on the Markov model (Fiering, 1971) are used to represent the inflows, R_n. Optimal releases obtained by solving the recursive relationship, Equation (12), are regressed on important variables to derive release policies. A detailed analysis of this method is given in study by the authors (1980) and Bhaskar (1978).

OPTIMAL RELEASE POLICIES

Chance-Constrained Programming

The solution of Equation (10) requires assigning values to the known quantities. The minimum storage, m_i, is assumed to be equal to the dead storage to the reservoir capacity, C. Values for the minimum and maximum allowable releases, q_i and f_i, respectively, are restricted by the feasibility requirements of the constraints imposed on them. For $\lambda_i = 0$ or 1, summing constraints on the minimum and maximum release, Equations (10d) and (10e) yield:

$$\sum_{i=1}^{12} q_i \leq \sum_{i=1}^{12} r_i^{(1-\alpha_3)} \qquad (14a)$$

$$\sum_{i=1}^{12} f_i \geq \sum_{i=1}^{12} r_i^{\alpha_4} \qquad (14b)$$

Table 2 lists percentile inflows, r_i^{α}, in every month at selected probability levels, α. These values are obtained from the fitted theoretical cumulative probability distribution of the historical flows. Since one of the main purposes of Hoover Reservoir is to meet the water supply requirements,

TABLE 2. Monthly Percentile Flows, r_i^{α} (cfs).

Month	Nonexceedence Probability, α							
	0.98	0.95	0.90	0.80	0.20	0.10	0.05	0.02
January	3463.378	1808.043	1096.633	589.928	53.517	27.660	16.610	9.488
February	1669.034	1187.969	880.069	601.845	145.474	99,484	72.967	50.401
March	1274.106	972.627	772.784	589.928	200.337	148.413	119.104	90.017
April	1450.988	1032.770	749.945	518.013	125.211	85.627	63.434	44.701
May	804.322	544.572	379.935	249.635	49.402	31.817	22.198	15.029
June	1603.590	943.881	584.058	327.013	35.874	19.886	12.183	7.029
July	1118.787	601.845	336.972	168.174	12.183	6.050	3.456	1.822
August	323.759	160.774	85.627	39.646	2.117	0.980	0.538	0.267
September	101.494	53.517	29.371	14.154	0.905	0.440	0.242	0.123
October	94.632	54.598	33.116	18.541	1.916	1.067	0.657	0.387
November	1096.633	518.013	273.144	127.740	6.821	3.158	1.682	0.819
December	2697.281	1408.105	699.244	330.300	17.289	8.005	4.179	2.014

it is reasonable to set the minimum guaranteed flow, q_i, at its maximum value while satisfying the feasibility requirement in Equation (14a). Assuming the same value of q_i in all months, Equation (14a) reduces to

$$q_i = \sum_{i=1}^{12} r_i^{(1-\alpha_3)}/12 \qquad (15)$$

The percentile flows in Table 2 are used to derive Figure 1, which displays the minimum flow that can be guaranteed at different levels of reliability, α_3. It is noted that the condition imposed on the minimum guaranteed flow by Equation (15) above makes the constraints in Equation (10d) binding.

The maximum allowable release from Hoover Reservoir in any month, f_i, is large enough to satisfy Equation (14b) as a strict inequality. This implies that the constraints on the maximum release, Equation (10e), are not critical in the solution of the present chance-constrained programming problem and could therefore be eliminated. The following chance-constrained programming models are examined in this paper.

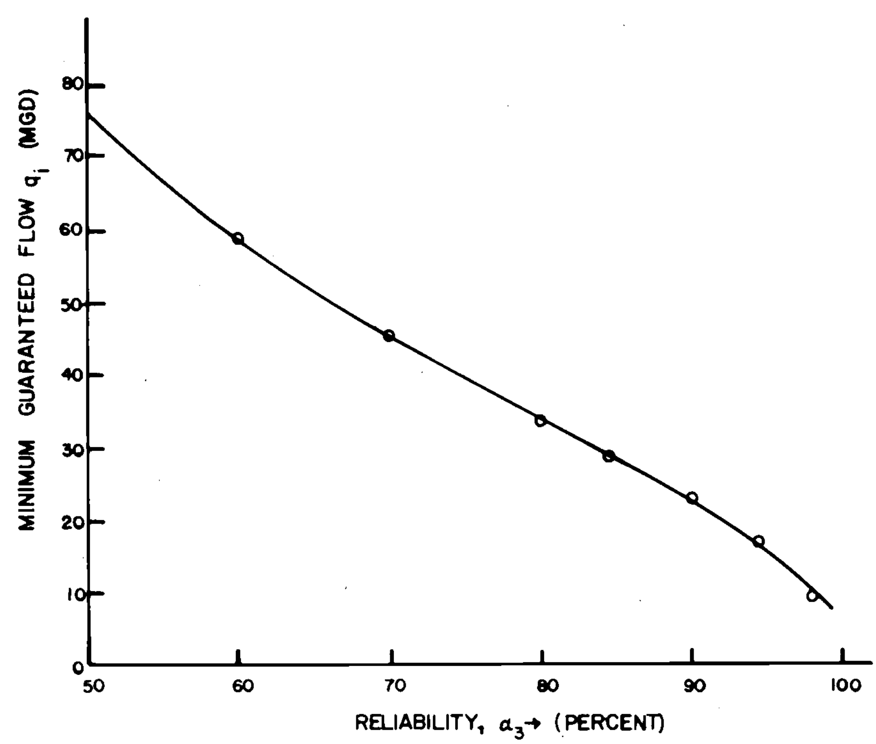

Figure 1. Minimum Guaranteed Flow, q_i, at Various Levels of Reliability, α_3.

Model LDR1. The chance-constrained linear program termed LDR1 in this study, is obtained by setting all λ_i equal to zero in Equations (10) and (11). It is solved by maintaining reliabilities α_1, α_2, and α_4 at a level of α^* while ranging α_3, the reliability on the minimum release constraint. These reliability levels are assumed to be equal in all months. Figure 2 illustrates the relationship between optimal reservoir capacity, reliability on the minimum guaranteed flow, and levels of reliability α^*. From Figures 1 and 2 it is observed that at the existing capacity of Hoover Reservoir (86,122 acre-feet), the reliability levels α^* and α_3 corresponding to a minimum guaranteed flow of 75 MGD are 0.83 and 0.50, respectively. The optimal monthly release policies derived under these conditions are presented in Table 3.

Model LDR2. Specifying a unit value for λ_i in Equations (10) and (11) yields the chance-constrained programming problem LDR2. This model differs from model LDR1 by including the current inflow, R_t, as an additional term in the linear decision rule (Equation 1). The relationship between the optimal reservoir capacity and the minimum guaranteed flow reliability, α_3, is shown in Figure 3. Unlike the model LDR1, this relationship is not a function of the reliability α^*. Also, the optimal reservoir capacity of 62,000 acre-feet required to meet a minimum guaranteed flow of 75 MGD at a reliability of 0.50 is less than the existing Hoover Reservoir capacity. Results indicate that the optimal release policies shown in Table 3 remain unchanged even if the existing Hoover Reservoir capacity is used in lieu of the optimal capacity.

Under similar parameter conditions, policies identical to those in Table 3 are obtained for both models, LDR1 and LDR2, when the expected value of the quadratic loss function, Equation (13), is substituted as the objective function in place of Equation (10a). This is shown in a previous study by Bhaskar (1978).

Dynamic Programming

Tables 3 and 4 present optimal monthly release policies derived by the dynamic programming-regression procedure. A synthetically generated inflow sequence of 50-year duration constitutes inflows into the reservoir for deriving the optimal releases using the Dynamic Programming algorithm. The reservoir is assumed to be full at the beginning and empty at the end operation. Observations from the initial and final period of operation are omitted, since for Hoover Reservoir the initial and terminal storage conditions only affect the optimal releases in the first and last year of operation, respectively. The forms of these policies are restricted to resemble the general form of the linear decision rule

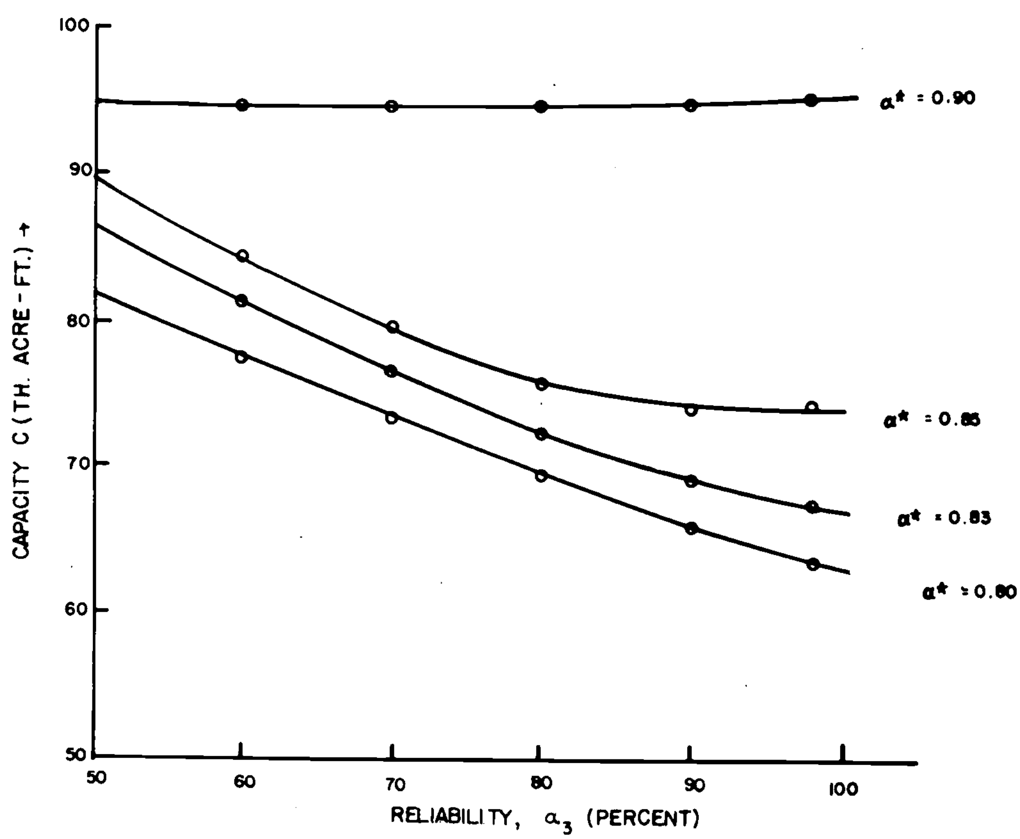

Figure 2. Required Reservoir Capacity, C, at Various Levels of Reliability on the Minimum Guaranteed Flow, q_i, for Model LDR1.

TABLE 3. Special Forms of Optimal Release Policies.*

Month	Model LDR1 $x_1=R_{i-1}+b_{i-1}-b_i$		Model LDR2 $X_i=R_i+b_{i-1}-b_i$		Model DP1 $X_i=B_0+B_1\ (R_{i-1})$			Model DP2 $X_i=B_0+B_1\ (R_i)$		
	b_i	$b_{i-1}-b_i$	b_i	$b_{i-1}-b_i$	B_0	B_1	ρ_1**	B_0	B_1	ρ_2***
January	−0.5786	2.5772	5.7351	−3.4980	5.1512	0.8395	0.715	2.3633	0.5178	0.880
February	2.9179	−3.4965	14.7825	−9.0474	7.5136	0.2479	0.752	2.7241	0.4924	0.643
March	11.9668	−9.0489	28.2736	−13.4911	5.3884	0.3811	0.573	4.8304	0.3628	0.693
April	25.4579	−13.4911	36.3613	−8.0877	5.3641	0.3076	0.722	6.3952	0.3139	0.584
May	33.5456	−8.0877	35.9176	0.4437	10.3197	0.0634	0.188	6.3433	0.4634	0.836
June	33.1019	0.4437	35.1269	0.7907	6.1919	0.4431	0.830	5.9305	0.4402	0.876
July	32.3112	0.7907	30.7060	4.4209	7.2251	0.2706	0.753	7.4973	0.5048	0.787
August	27.8903	4.4209	24.0677	6.6383	8.5083	0.2830	0.640	9.3153	0.2885	0.568
September	21.2520	6.6383	17.0828	6.9849	9.3160	0.1745	0.421	9.6282	0.1460	0.158
October	14.2671	6.9849	10.2619	6.8209	9.8698	0.1340	0.145	9.7537	0.4259	0.148
November	7.4462	6.8209	4.8143	5.4476	9.4184	0.9547	0.278	8.0988	0.4819	0.704
December	1.9986	5.4476	2.2371	2.5772	4.6488	1.8615	0.741	5.4972	0.5951	0.890

*Existing Hoover Reservoir design is assumed, all units in thousand acre-feet, target=75 MGD.
**Correlation of release with the previous month's inflow.
***Correlation of release with the current month's inflow.

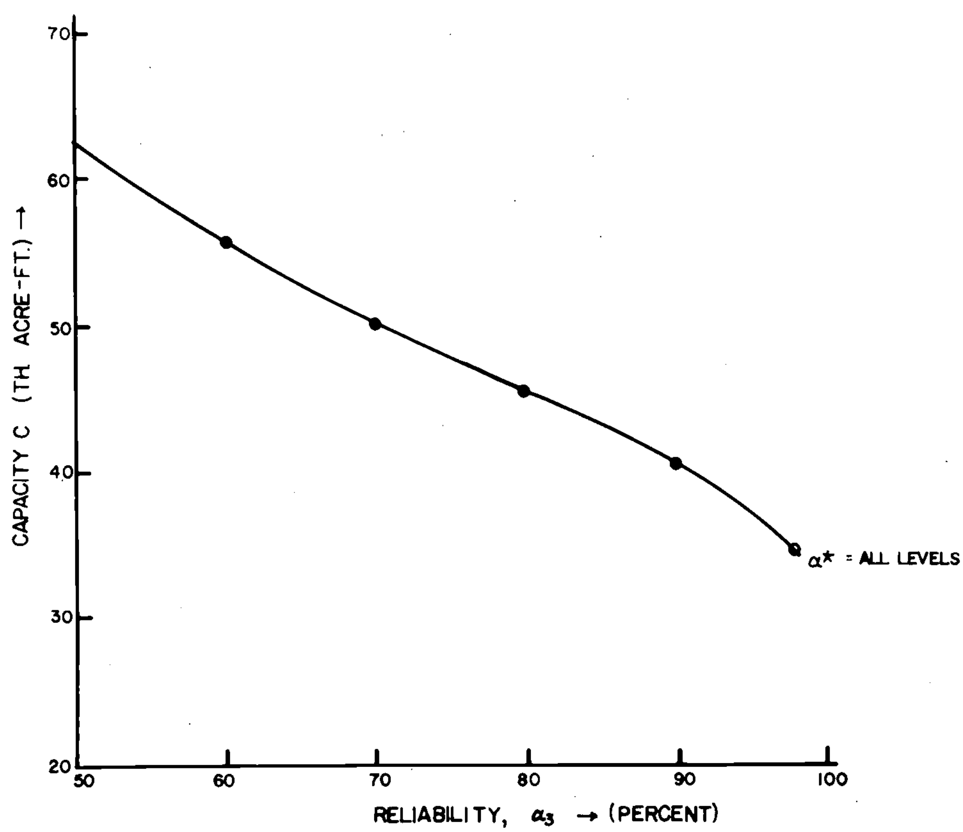

Figure 3. Required Reservoir Capacity, C, at Various Levels of Reliability on the Minimum Guaranteed Flow, q_i, for Model LDR2.

TABLE 4. General Linear Release Policy.*

Month	Model DP3: $X_t = B_0 + B_1(R_t) + B_2(R_{t-1})$						
	B_0	B_1	ρ_1**	B_2	ρ_2**	R	ρ_3**
January	1.3422	0.4224	0.880	0.2945	0.715	0.900	0.647
February	1.8665	0.3204	0.643	0.1958	0.752	0.846	0.378
March	2.0844	0.2850	0.693	0.2171	0.573	0.749	0.464
April	3.5834	0.1611	0.584	0.2443	0.722	0.768	0.496
May	5.0339	0.4634	0.835	0.0634	0.136	0.846	0.000
June	5.6228	0.3057	0.876	0.1694	0.830	0.892	0.843
July	6.9323	0.3271	0.788	0.1341	0.753	0.826	0.744
August	8.5642	0.1270	0.567	0.2089	0.640	0.666	0.672
September	9.1489	0.1207	0.157	0.1709	0.420	0.439	0.066
October	9.6516	0.3495	0.149	0.1076	0.144	0.189	0.232
November	8.1385	0.4909	0.704	--0.1050	0.277	0.704	0.429
December	6.0583	0.6832	0.890	−0.3793	0.741	0.893	0.873

*Existing Hoover Reservoir design is assumed, all units in thousand acre-feet, target=75 MGD.
**ρ_1 and ρ_2 are correlation coefficients of monthly release, X_t with the current and previous month's inflows, R_t and R_{t-1}, respectively. ρ_3 is the correlation between the inflows R_t and R_{t-1}. These estimates are based on a sample size of 148.

(Equation 5) and are discussed in the following section. This is done to examine the performance of the linear decision rule under a wider range of possible values for the parameter λ_i (refer to Equations 6 through 8). An even greater variety of policies possible under the dynamic programming-regression approach are presented in Bhaskar and Whitlatch (1980).

COMPARISON OF OPTIMAL RELEASE POLICIES

Tables 3 and 4 summarize, for the purpose of comparison, special forms of optimal monthly release policies derived under the chance-constrained linear programming and dynamic programming-regression methodologies. These policies are derived for Hoover Reservoir having a mean detention time of 0.44 years and a target water supply release of 75 MGD. Similar policies, not presented here, are derived at other capacity-target levels. These policies can be mathematically defined as follows:

Model LDR1 (refer to Equation 5 with $\lambda_i = 0$)

$$X_t = (b_{i-1} - b_i) + R_{t-1} \tag{16a}$$

Model LDR2 (refer to Equation 5 with $\lambda_i = 1$)

$$X_t = (b_{i-1} - b_i) + R_t \tag{16b}$$

Model DP1 $$X_t = B_0 + B_1 R_{t-1} \tag{16c}$$

Model DP2 $$X_t = B_0 + B_1 R_t \tag{16d}$$

Model DP3 $$X_t = B_0 + B_1 R_t + B_2 R_{t-1} \tag{16e}$$

where B_0, B_1, and B_2 are regression coefficients. Models LDR1 and LDR2 release policies would be similar in form to Models DP1 and DP2, respectively, if the regression coefficients B_0 and B_1 are set equal to $(b_{i-1} - b_i)$ and unity, respectively. Thus, the chance-constrained policies are more restrictive than the dynamic programming policies since they assign a value of unity to the coefficient of the inflow terms in the above release policies.

Since the chance-constrained policies implicitly assume a two-sided quadratic loss function for a given reservoir capacity, C (i.e., optimal policies do not change by using this as the objective function in lieu of reservoir capacity, C, in Equation 10), the dynamic programming policies are derived using a similar loss function. Results show that:

a) unlike the chance-constrained linear release policies in models LRD1 and LRD2, the coefficients associated with the current or previous periods inflow in the policies DP1 and DP2, are not equal to unity, and in general are widely different from unity; and

b) the intercept terms, B_0, in policies DP1 and DP2 do not correspond to the related terms $(b_{i-1} - b_i)$, in policies derived under models LDR1 and LDR2.

The above conclusions hold at all target levels, since both the chance-constrained and dynamic programming results are target independent under a two-sided quadratic loss function (Bhaskar, 1978). The same conclusions have been verified at other reservoir capacities as well.

Simulation of Release Policies Under Existing Design. The operation of Hoover Reservoir is simulated using the linear release policies shown in Tables 3 and 4. A synthetically generated inflow sequence of 148 year duration constitutes inflows into the reservoir. Overall performance measures, based on 20 such simulations, are evaluated to compare operational differences inherent in the release policies.

Statistics on the average percentage of shortages, presented in Table 5, reflect the reliability with which the target release can be met in each month. It may be recalled from the previous section that the chance-constrained linear programming models LDR1 and LDR2 were solved assuming a 50 percent reliability of satisfying the target release (75 MGD). Results in Table 5 indicate that, at a similar target-level of 75 MGD, the dynamic programming policies also exhibit percentage monthly shortages which are within the 50 percent reliability level.

TABLE 5. Average Monthly Shortages* (percent).

Month	LDR1	DP1	LDR2	DP2	DP3
January	30.78	17.43	35.27	29.87	28.99
February	33.14	1.05	38.58	4.53	10.54
March	41.12	0.78	48.78	1.59	6.45
April	47.50	1.00	41.96	0.17	2.33
May	42.37	0.78	35.88	1.79	4.29
June	34.60	4.05	34.87	6.72	6.32
July	37.13	7.70	30.91	3.85	1.69
August	32.80	15.54	21.01	10.37	6.12
September	15.51	25.68	0.10	20.44	15.71
October	23.55	39.66	29.53	35.10	31.11
November	25.75	43.01	24.70	39.39	37.94
December	34.73	38.89	31.35	29.93	28.85

*Based on actual releases short of the target (75 MGD) with existing Hoover Reservoir design assumed.

Results for the average monthly releases, storages, and losses (measured using a two-sided quadratic loss function) suggest that the chance-constrained programming policies of models LDR1 and LDR2 give values and trends which are quite different from those under the dynamic programming policies, DP1 and DP2. Table 6 indicates that the average annual loss obtained using the chance-constrained programming policies yield losses which are almost 60 percent higher than the corresponding dynamic programming policies. On the other hand, a comparison between the policies DP1, DP2, and DP3 show average losses that are in close agreement. The loss under the standard operating policy is also included for illustrative purposes, although the nature of this policy would be similar to the operation of a reservoir under a one-side quadratic loss function that penalizes releases falling short of the target only. Releases in excess of the target are assigned no loss. A more detailed comparison of the standard policy with release policies derived using dynamic programming under a one-sided quadratic loss function is given in Bhaskar (1978). Results show that release policies derived using dynamic programming yield lower losses.

TABLE 6. Average Annual Loss Per Year.*

Model					
LDR1	DP1	LDR2	DP2	DP3	Standard Policy
1987.16	1246.45	1835.41	1155.61	1138.67	1786.79

*Under a two-sided quadratic loss function; existing Hoover Reservoir capacity of 86,123 acre-feet; target release of 75 MGD.

SUMMARY CONCLUSIONS AND RECOMMENDATIONS

Monthly release policies are derived for a single, multipurpose reservoir using chance-constrained linear programming and dynamic programming-regression methodologies. A comparison of monthly release policies using these two mathematical optimization techniques, and the search for an appropriate form of a monthly release policy is the major emphasis of this study. Simulation procedures, in conjunction with operational hydrology, are used to measure and verify the performance of the release policies. Three conclusions are made as a result of this study.

First, unlike the chance-constrained linear programming policies, LDR1 and LDR2 (Equations 16a and 16b), the coefficient associated with the current or previous period's inflow in the dynamic programming policies, DP1 and DP2 (Equations 16c and 16d) is not equal to unity. This is true at all target levels and mean detention times.

Second, at a mean detention time of 0.44 years (existing Hoover Reservoir design) both the derived chance-constrained programming and dynamic programming policies satisfy the reliability, α_3, of meeting the minimum guaranteed flow, q_i. In addition, the derived dynamic programming policies produce a lower average annual loss while achieving at least as high reliability levels. This observation would argue for more research on deriving generalized linear (or nonlinear) decision rules where in the present and past period's inflows are weighted in an optimal manner.

Third, the use of the general form of the linear decision rule, $X_t = b + \lambda_1 R_t + \lambda_2 R_{t-1} + \ldots + \lambda_n R_{t-n-1}$, requires evaluation of the optimal values of the coefficients, b, λ_1, $\lambda_2 \ldots \lambda_n$. Consequently, there is a need to develop methods to determine these coefficients explicitly. The dynamic programming-regression approach discussed here may provide valuable information to assist in the search for these optimal coefficients.

ACKNOWLEDGMENTS

The work upon which the paper is based was supported in part by funds provided by the United States Department of the Interior, Office of Water Research and Technology, as authorized by the Water Resources Act of 1964.

LITERATURE CITED

Bhaskar, N. R., 1978. Application of Mathematical Optimization Techniques in Reservoir Design and Management Studies. Ph.D. Dissertation, The Ohio State University, 259 pp.

Bhaskar, N. R. and E. E. Whitlatch, Jr., 1980. Derivation of Monthly Reservoir Release Policies. Water Resources Research 16(6): 987-993.

Fiering, M. B. and B. Jackson, 1971. Synthetic Streamflows. Water Resources Monograph Number 1, American Geophysical Union, Washington, D.C., 98 pp.

Loucks, D. P., 1970. Some Comments on Linear Decision Rules and Chance-Constraints. Water Resources Research 6(2):668-671.

Loucks, D. P. and P. J. Dorfman, 1975. An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation. Water Resources Research 11(6):777-782.

ReVelle, C. and J. Gundelach, 1975. Linear Decision Rule in Reservoir Management and Design 4. A Rule That Minimizes Output Variances. Water Resources Research 11(2):197-207.

ReVelle, C., E. Joeres, and W. Kirby, 1969. The Linear Decision Rule in Reservoir Management and Design 1, Development of the Stochastic Model. Water Resources Research 5(4):767-777.

Stedinger, J. R., B. F. Sule, and D. P. Loucks, 1984. Stochastic Dynamic Programming Models for Reservoir Operation Optimization. Water Resources Research 20(11):1499-1505.

Yeh, W. W-G., 1985. Reservoir Management and Operations Models: A State-of-the-Art Review. Water Resources Research 21(12): 1797-1818.

Young, G. K., 1966. Techniques for Finding Reservoir Operating Rules. Ph.D. Dissertation, Harvard University, Cambridge, Massachusetts, 89 pp.